Behavioral Network Economics

A Cumulative Prospect Theoretic Perspective

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Dissertation Talk

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Network Economics

Study of problems where several agents interact with each other over a network with shared resources

Previous line of work

- **Optimal routing**:
  shortest path, TSP, min cost max flow

- **Network formation and growth**:
  random network models, spread of information and diseases, connectivity and segregation

Network Economics

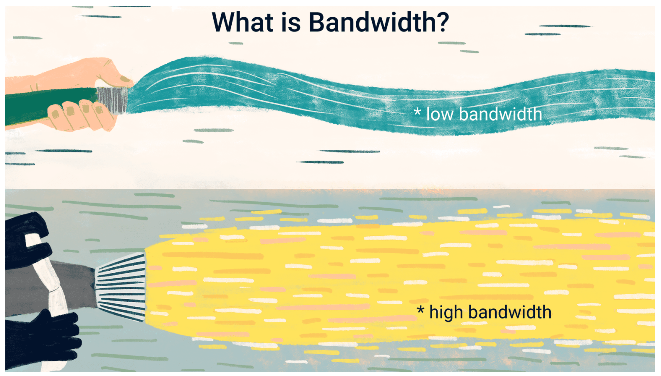
Study of problems where several agents interact with each other over a network with shared resources

An Emerging Paradigm

- Centralized control
- Direct-to-consumer services
- Increasing effect of human interactions and decisions
- Limited resources. Expanding the resources is not always possible

How to allocate resources over these networks while taking user behavior into account?

Network Economics Today

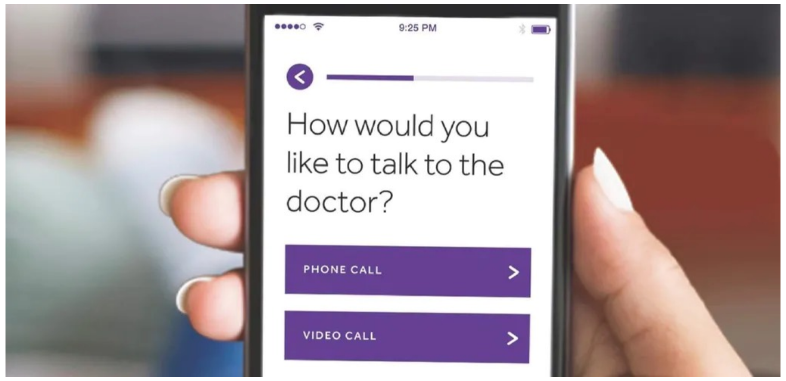


Broadband network

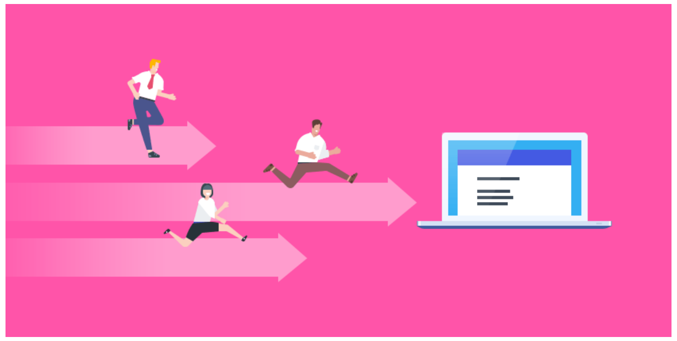
Cloud Computing

Smart Grid

Ad Auctions



Telemedicine



Labor Markets

How to incorporate human preferences?

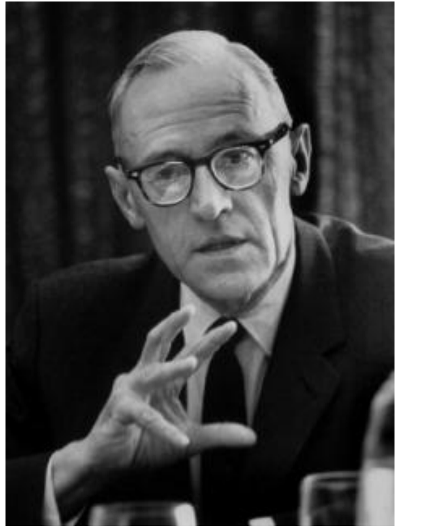
We seek market-based solutions e.g. auctions and fixed rate pricing.

This Talk

- Capture human behavior using sophisticated models from psychology
  - Tool: Cumulative Prospect Theory
- Build mathematical models to provide theoretical insights and facilitate automation
  - Network resource allocation model
  - Market-based mechanism and a pricing scheme
- Other Avenues:
  - Scheduling in Cloud Computing and Pricing
  - Mechanism Design
- Future Directions
  - Using ML/AI components as a part of these markets

Expected Utility Theory (EUT)

(Von Neumann-Morgenstern 1947)



Lottery

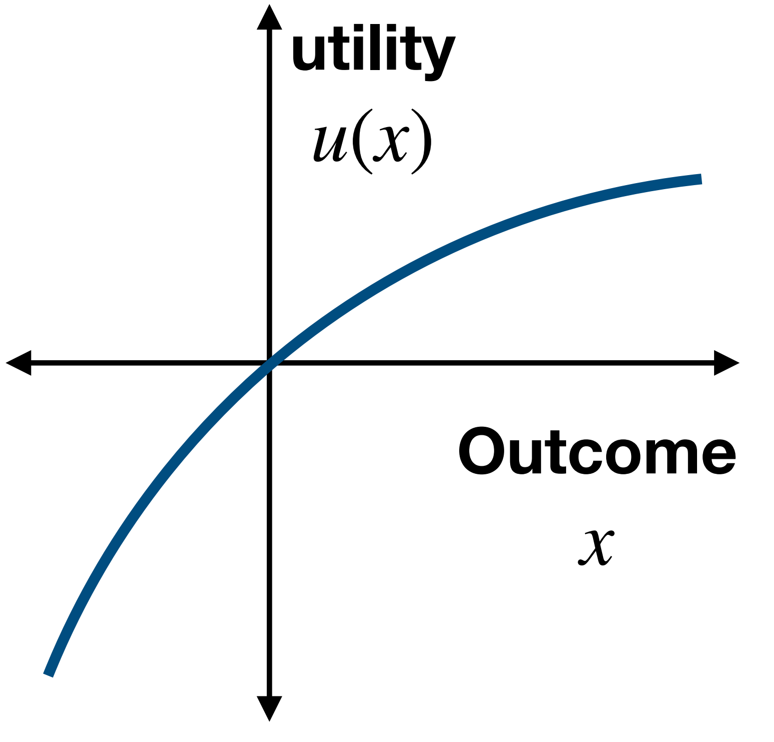
$L =$

probability	0.1	0.2	0.15	0.1	0.25	0.2
outcome	10	5	2	0	-1	-3

Expected utility of lottery L

$$U(L) = 0.1u(10) + 0.2u(5) + 0.15u(2) + 0.1u(0) + 0.25u(-1) + 0.2u(-3)$$

Lottery with higher Expected Utility is preferred.



Utility function

Allais Paradox (1953)

Allais

Experiment 1			
Gamble 1A		Gamble 1B	
Winnings	Chance	Winnings	Chance
$1 million	100%	$1 million	89%
		Nothing	1%
		$5 million	10%

Experiment 2			
Gamble 2A		Gamble 2B	
Winnings	Chance	Winnings	Chance
Nothing	89%	Nothing	90%
$1 million	11%		
		$5 million	10%

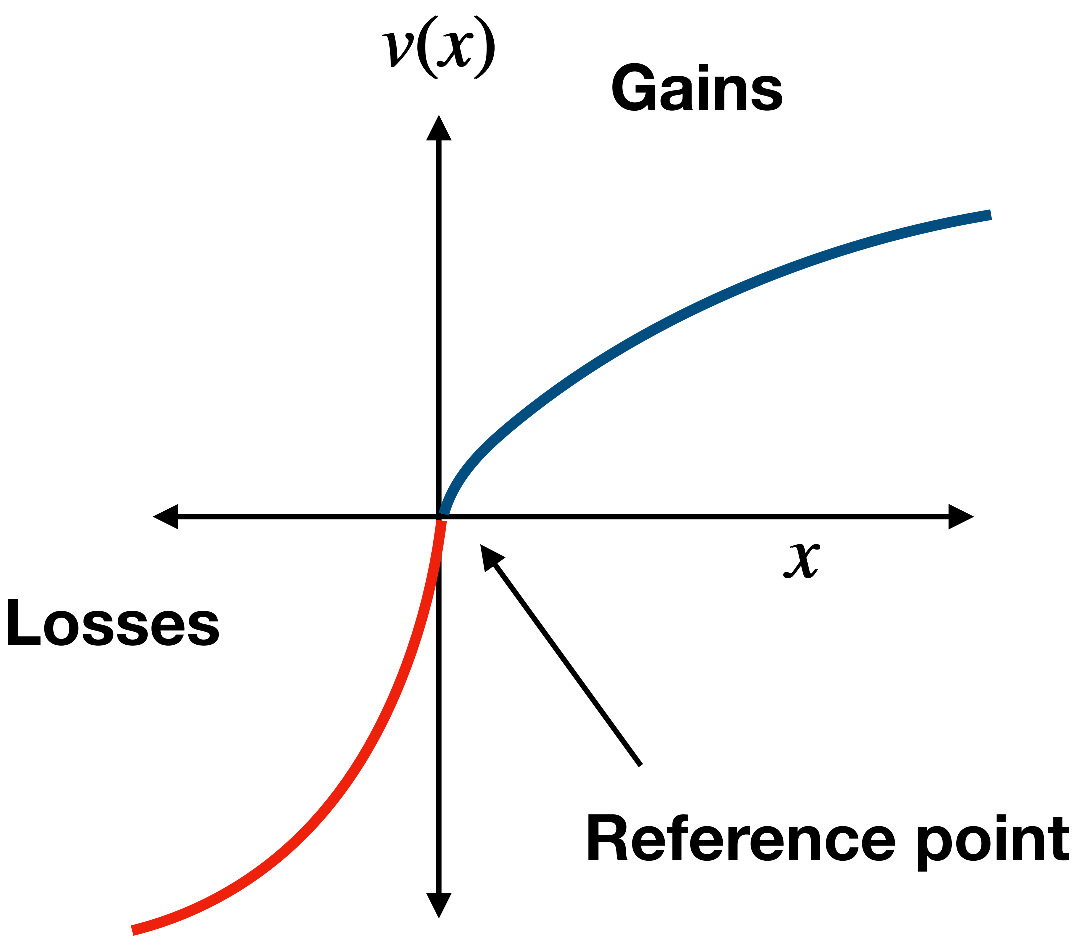
People often do NOT follow EUT!

Cumulative Prospect Theory (CPT)

(Kahneman - Tversky 1992)

Cumulative Prospect Theory (CPT)

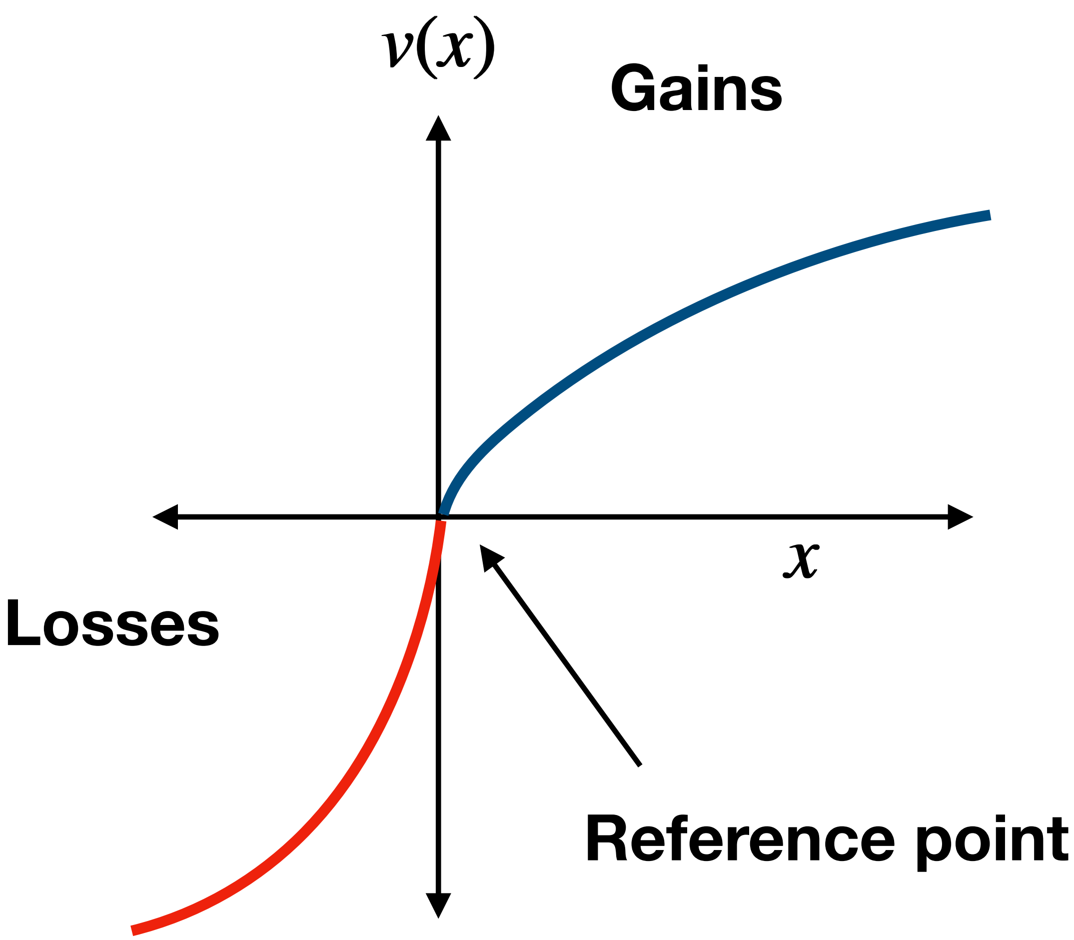
(Kahneman - Tversky 1992)



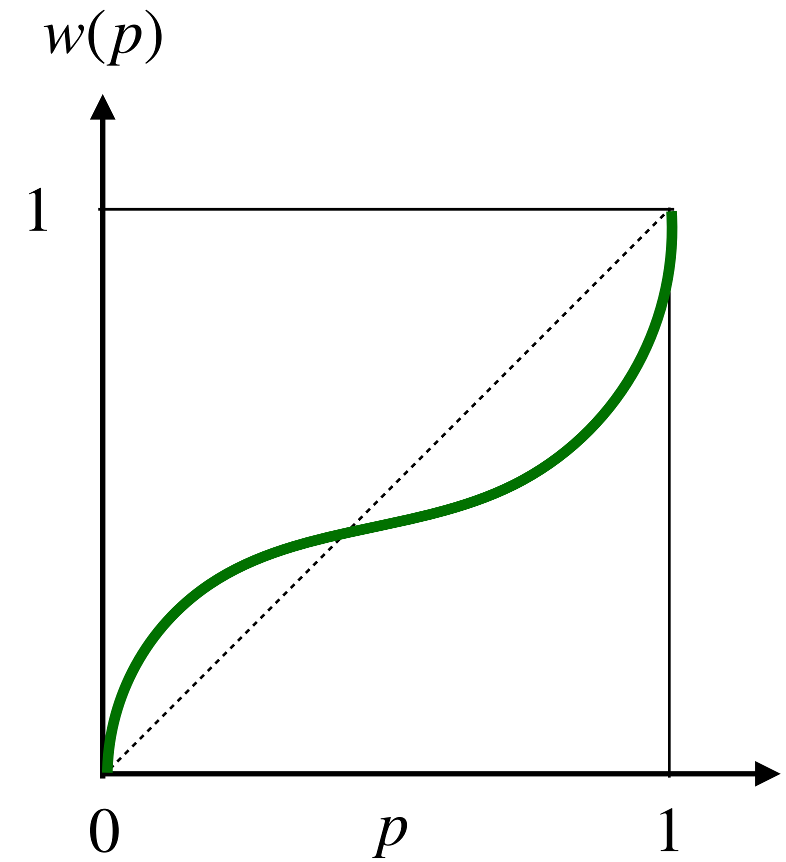
Value Function

Cumulative Prospect Theory (CPT)

(Kahneman - Tversky 1992)



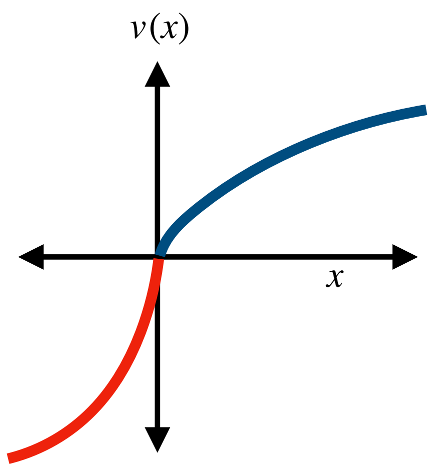
Value Function

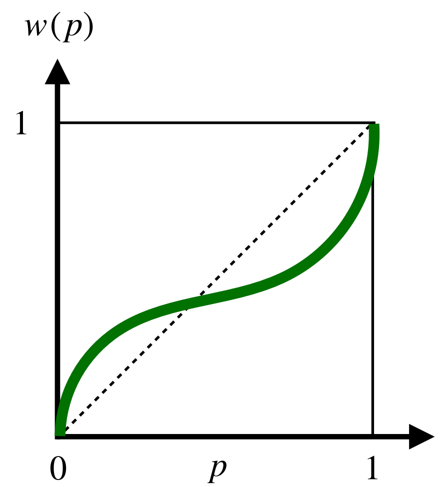


Probability Weighting Function

Cumulative Prospect Theory (CPT)

(Kahneman - Tversky 1992)





Suppose $r = 0$

$L =$

	Gain				Loss	
probability	0.1	0.2	0.15	0.1	0.25	0.2
outcome	10	5	2	0	-1	-3

CPT value of Lottery L

$$V(L) = V^{gain}(L) + V^{loss}(L)$$

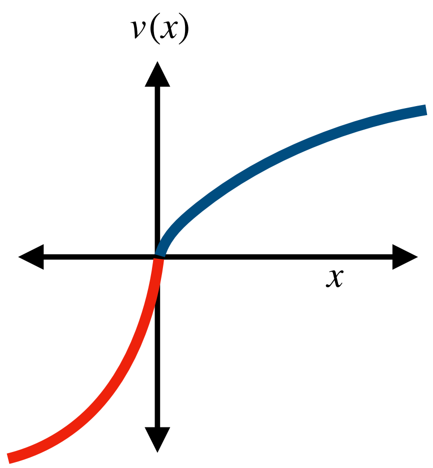
$$V^{gain}(L) = v(10)[w^+(0.1)] + v(5)[w^+(0.1+0.2) - w^+(0.1)] + v(2)[w^+(0.1+0.2+0.15) - w^+(0.1+0.2)]$$

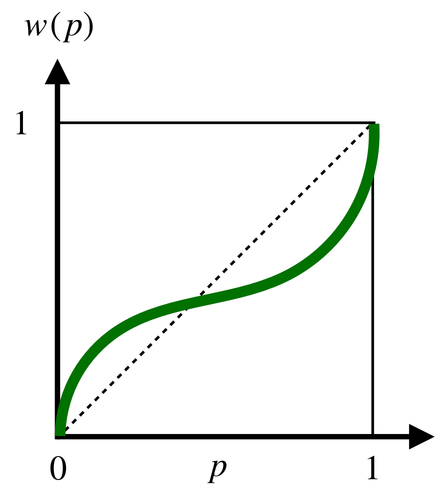
$$V^{loss}(L) = v(-3)[w^-(0.2)] + v(-1)[w^-(0.2+0.25) - w^-(0.2)]$$

Cumulative Prospect Theory (CPT)

(Kahneman - Tversky 1992)





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	Gain				Loss	
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outcome	10	5	2	0	-1	-3

First Order Stochastic Dominance Fails!

CPT value of Lottery L

Why not $w(0.2)$?

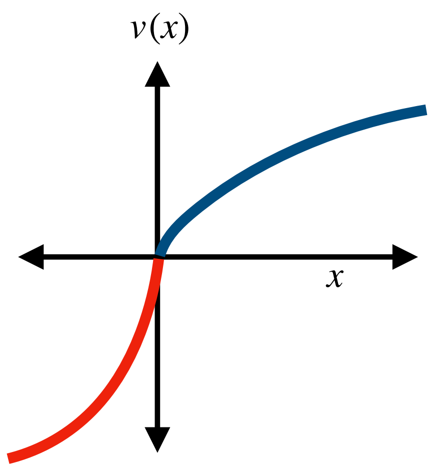
$$V(L) = V^{gain}(L) + V^{loss}(L)$$

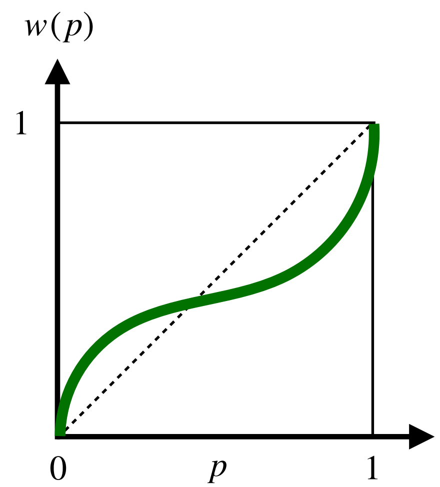
$$V^{gain}(L) = v(10)[w^+(0.1)] + v(5)[w^+(0.1+0.2) - w^+(0.1)]$$
$$+ v(2)[w^+(0.1+0.2+0.15) - w^+(0.1+0.2)]$$

$$V^{loss}(L) = v(-3)[w^-(0.2)] + v(-1)[w^-(0.2+0.25) - w^-(0.2)]$$

Cumulative Prospect Theory (CPT)

(Kahneman - Tversky 1992)





Rank Dependence
Ordering in outcomes affects the corresponding decision weights

CPT generalizes EUT!

Suppose $r = 0$

$L =$

	Gain				Loss	
probability	0.1	0.2	0.15	0.1	0.25	0.2
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CPT value of Lottery L

$$V(L) = V^{gain}(L) + V^{loss}(L)$$

$$V^{gain}(L) = v(10)[w^+(0.1)] + v(5)[w^+(0.1+0.2) - w^+(0.1)]$$
$$+ v(2)[w^+(0.1+0.2+0.15) - w^+(0.1+0.2)]$$

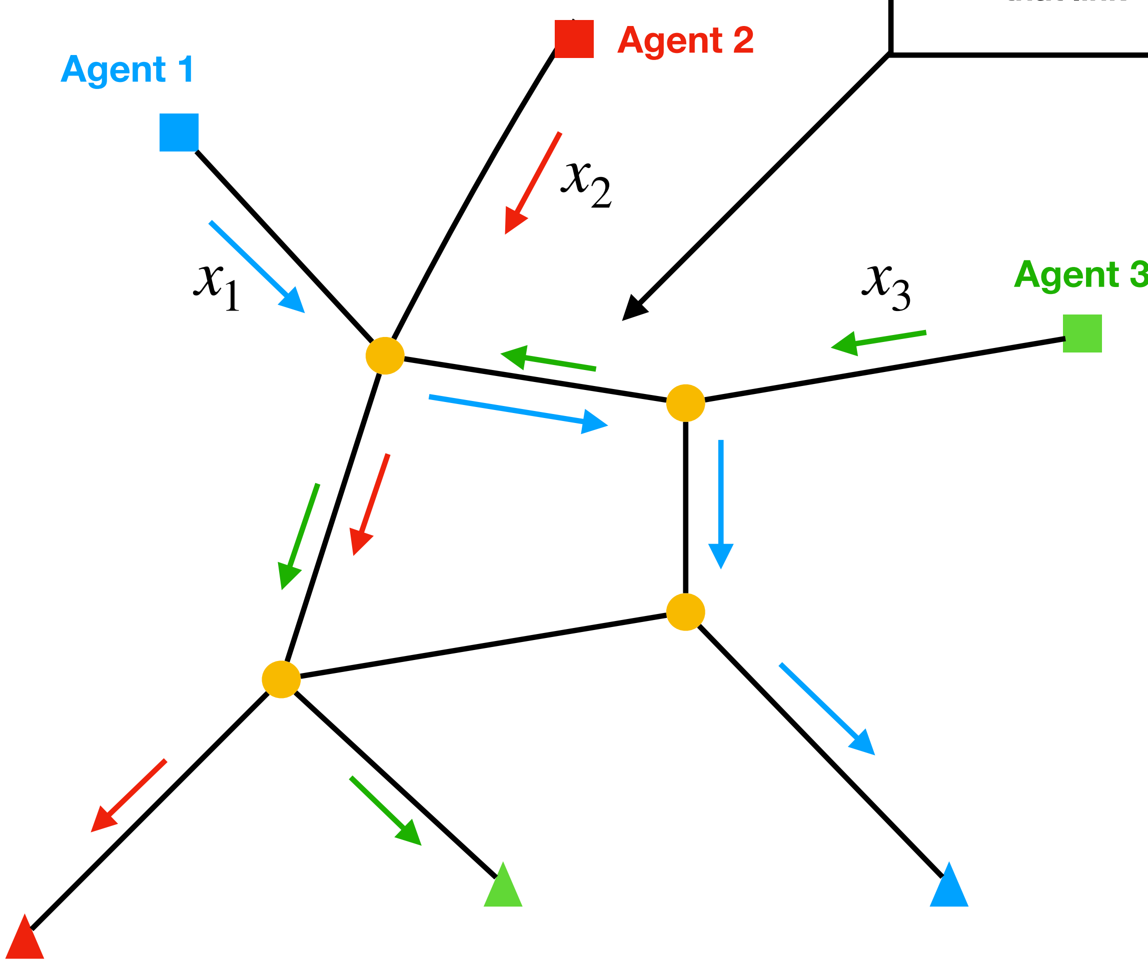
$$V^{loss}(L) = v(-3)[w^-(0.2)] + v(-1)[w^-(0.2+0.25) - w^-(0.2)]$$

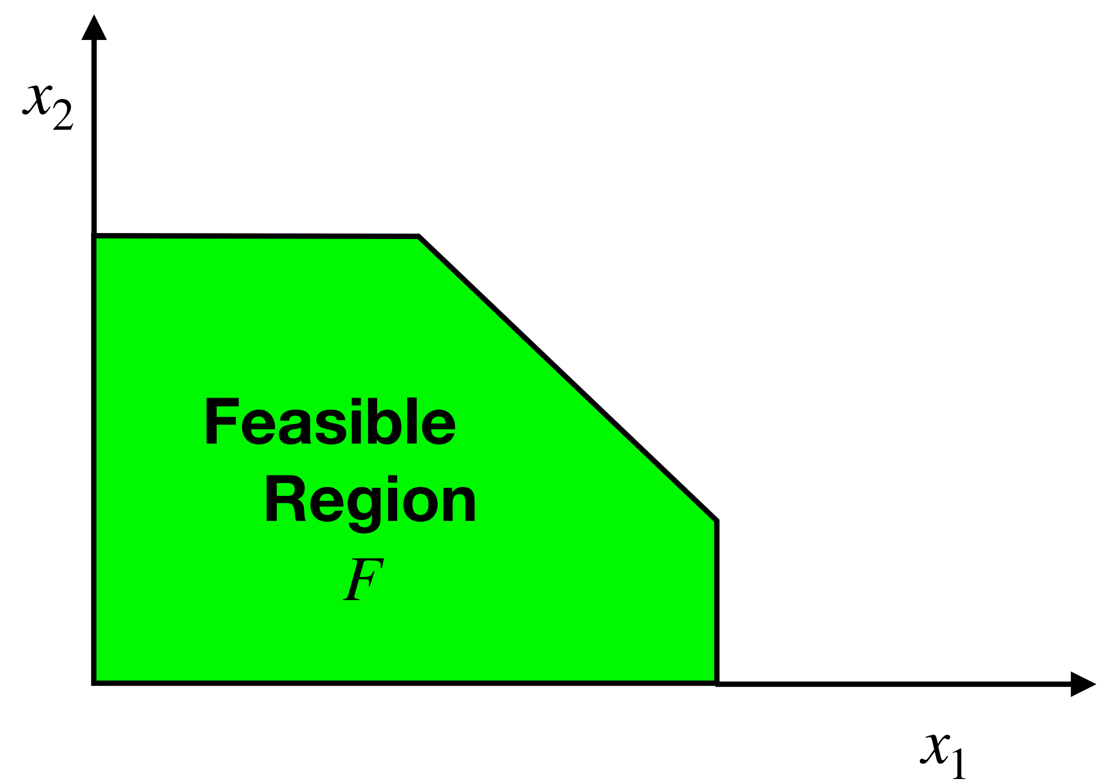
Kelly Network Model

Bandwidth Allocation over the Internet

Link Capacity Constraint

$$\sum_{\text{Agents using that link}} x_i \leq c_j$$



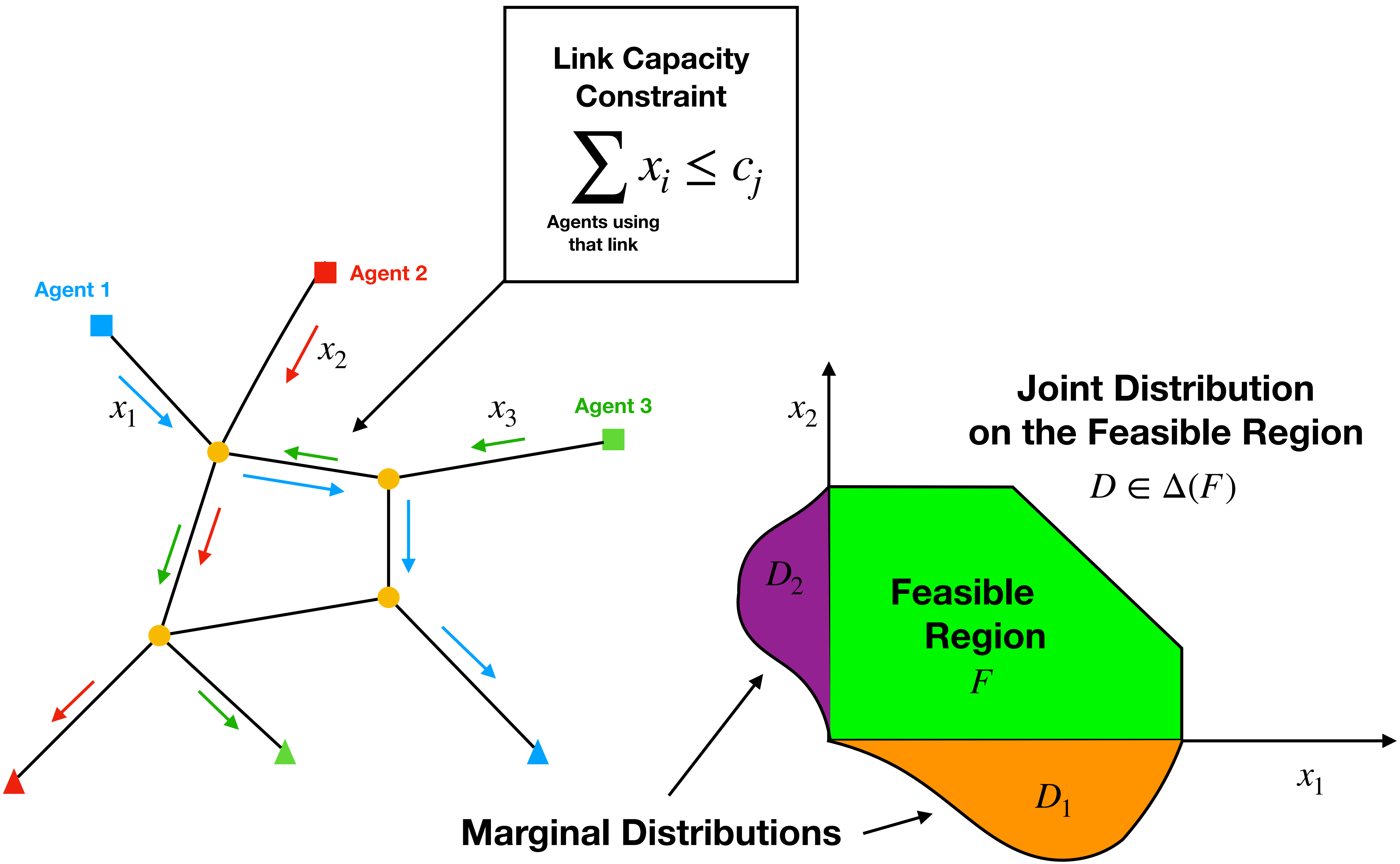


Deterministic vs Randomized Allocations

- This literature has mainly focused on deterministic allocations.
- Quality of Service has another very important aspect: uncertainty in the allocations
- Different users have different preferences towards uncertainty
- Do randomized allocations (lotteries) provide an advantage over deterministic allocations?
- If yes, does there exist a market-based mechanism to implement optimum lottery?

Network Model

Bandwidth Allocation over the Internet



System Problem

CPT value of Agent i's Lottery

Maximize $\sum_i V_i(L_i)$ Total Social Welfare

subject to $D \in \Delta(F)$ Probability Distribution on Feasibility Region

$D_i = L_i, \forall i$ Marginal Distributions agree with Lotteries

- System Operator (like Comcast) does not know agent's CPT features, namely, the value function and the probability weighting function
- Goal: Design market-based mechanism to solve it.

System Problem

Deterministic

Maximize $\sum_i v_i(x_i)$

subject to $x \in F$

Market-based solution (Kelly 1998)

- Convex optimization problem
- Facilitates decomposition of this problem into several agent problems and one system operator problem
- Underlies TCP/IP protocol
- Achieves optimum solution at equilibrium

Randomized

Maximize $\sum_i V_i(L_i)$

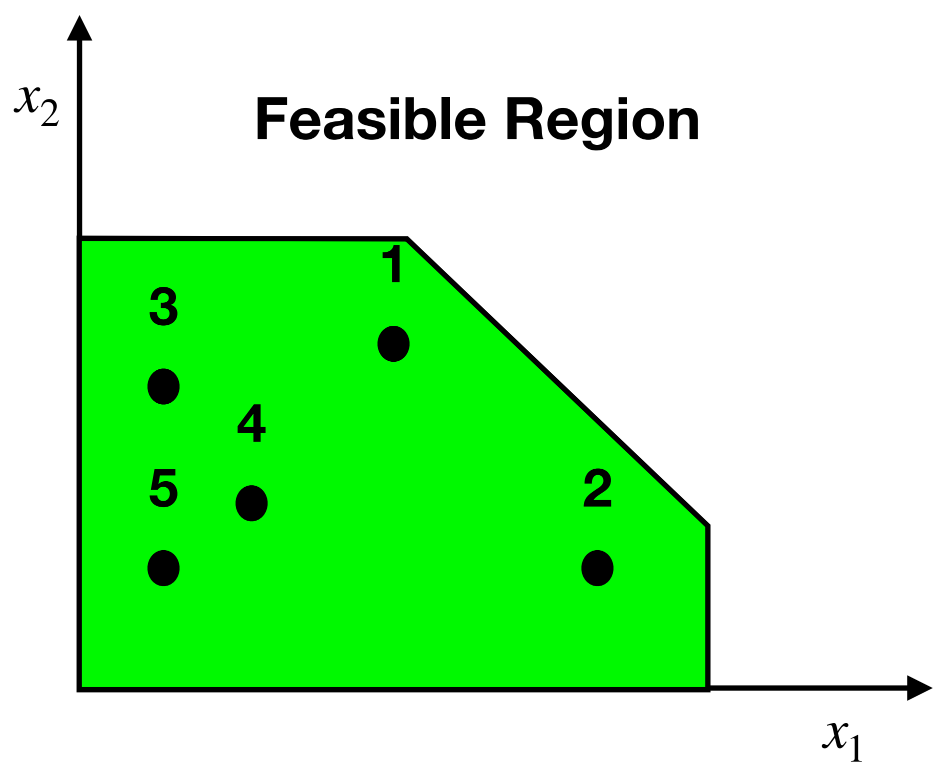
subject to $D \in \mathscr{P}(F)$

$$D_i = L_i, \forall i$$

- Non-Convex because of prob. weight. func.
- Rank dependence introduces additional complexity

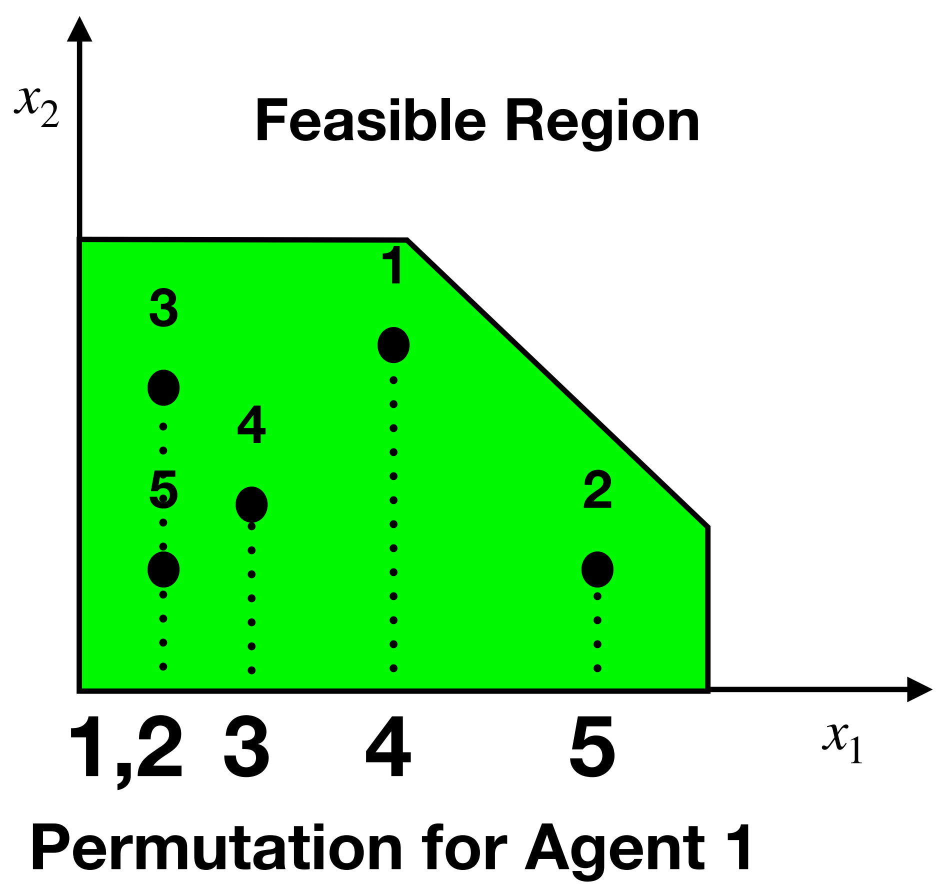
Discretization

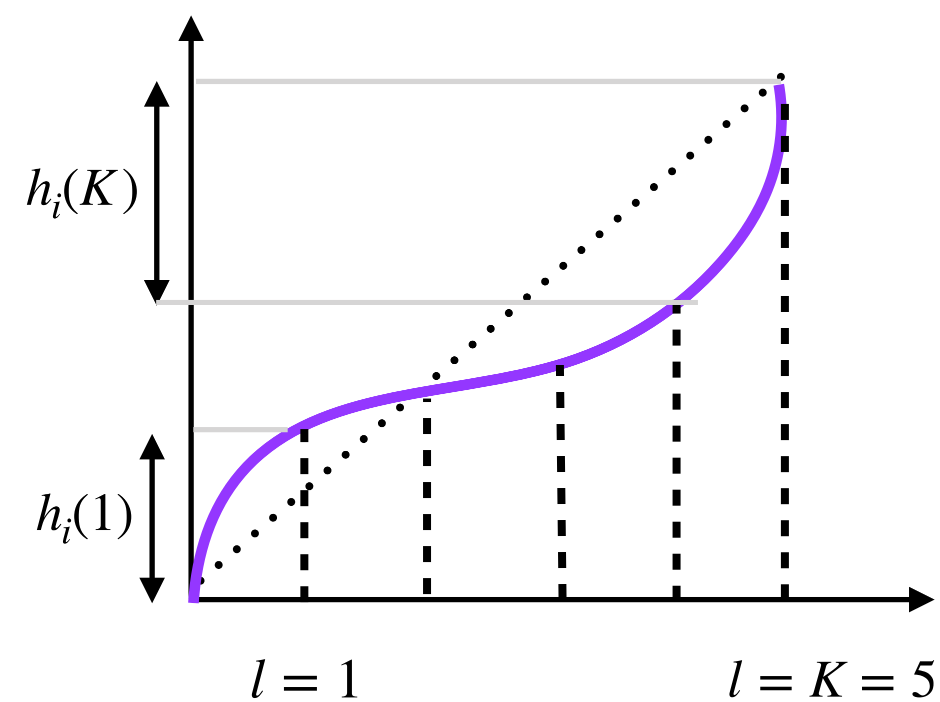
- A simplification that goes a long way …
- Fix a positive integer K and discretize the joint distribution D into K alternatives, say, 1, 2, …, l, …, K.
- One alternative is chosen uniformly at random from these

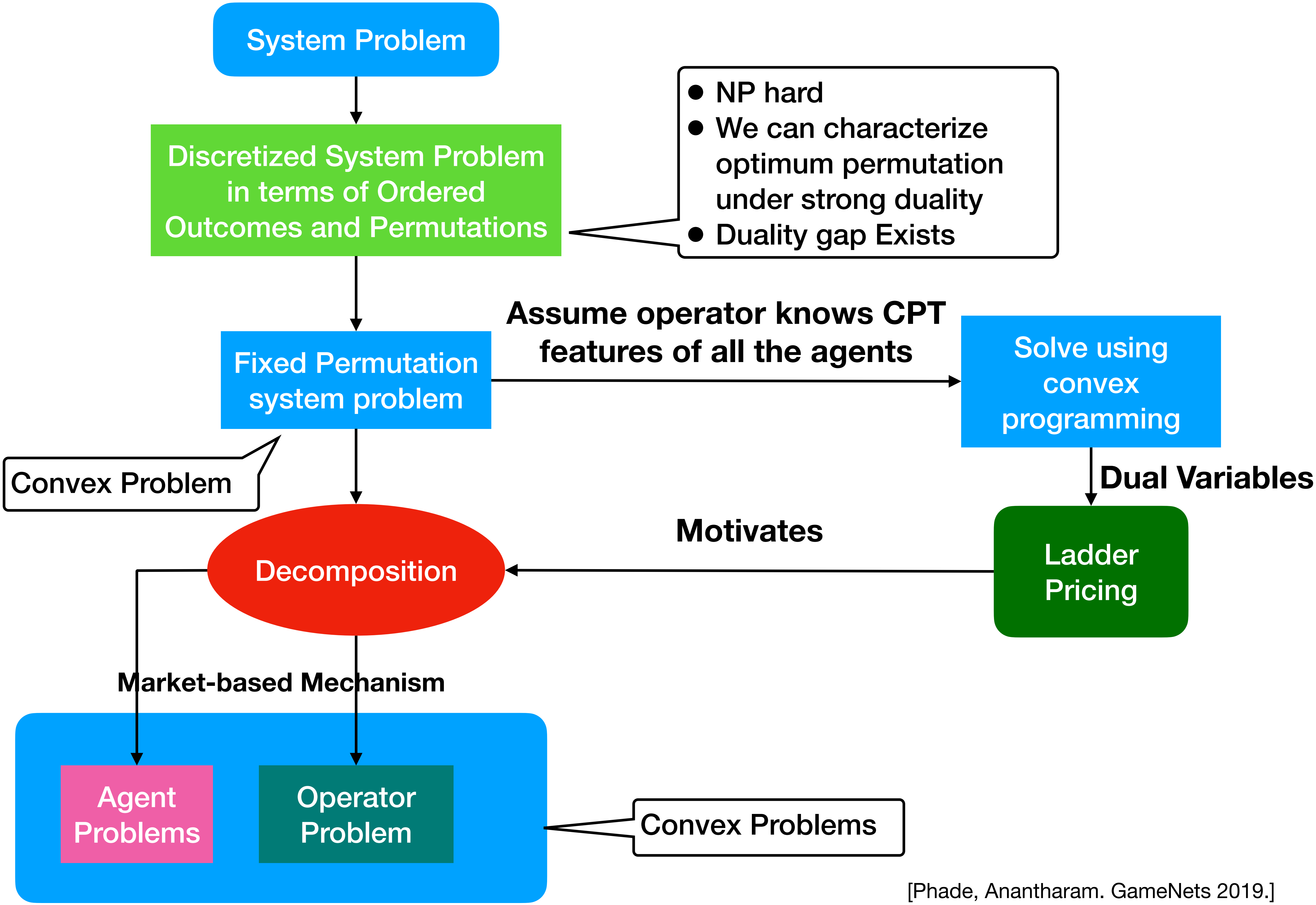


Discretization

- Restrict attention to probability weighting jumps
- Separate the permutation structure in the problem







[Phade, Anantharam. GameNets 2019.]

Ladder Pricing

Budget Order — Place Your Order Here

	0	0.2	0.4	0.6	0.8	1
Rate Card	Base $10	1st Boost $5	2nd Boost $3	3rd Boost $2	4th Boost $1	

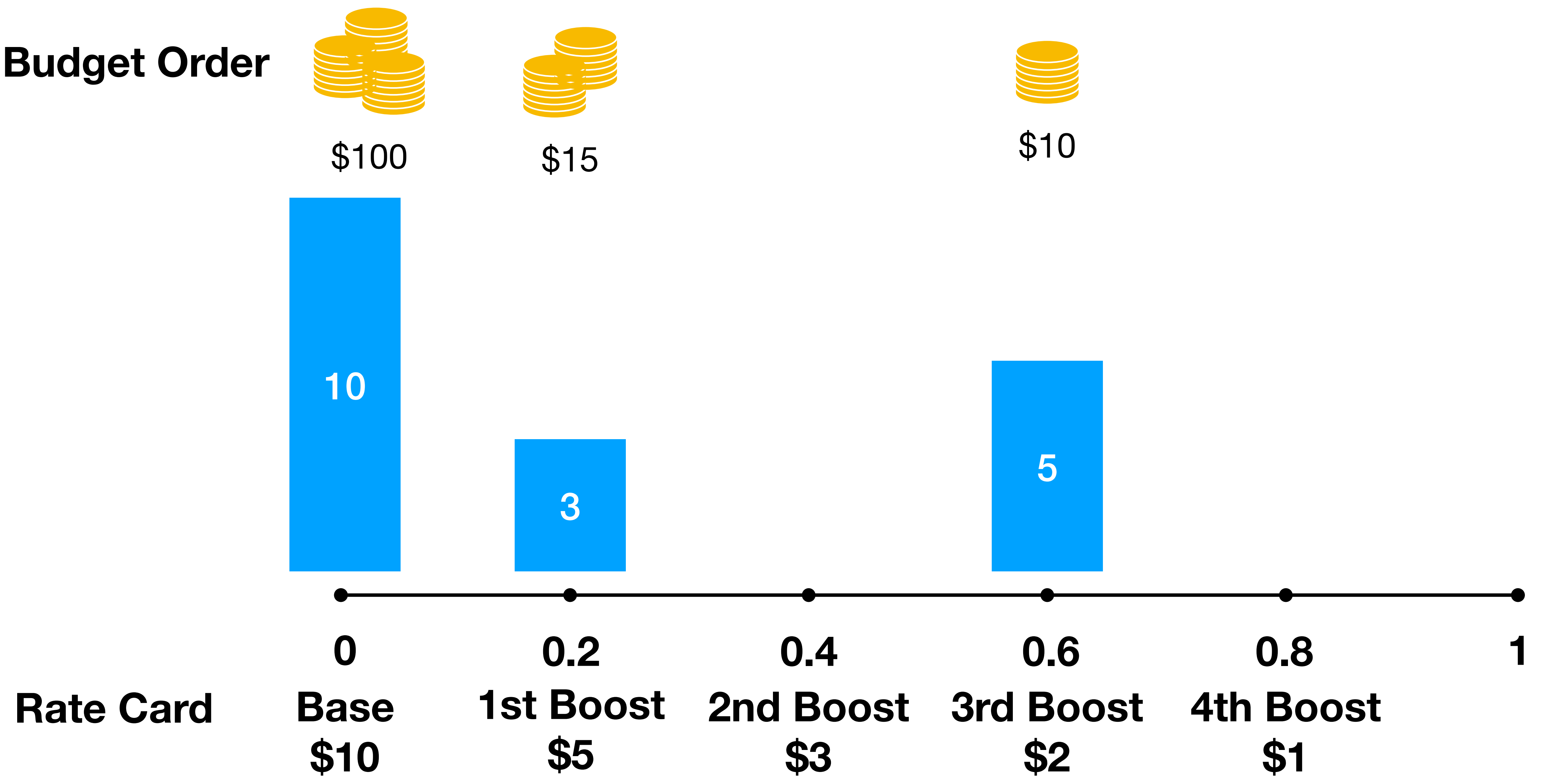
Ladder Pricing

Budget Order

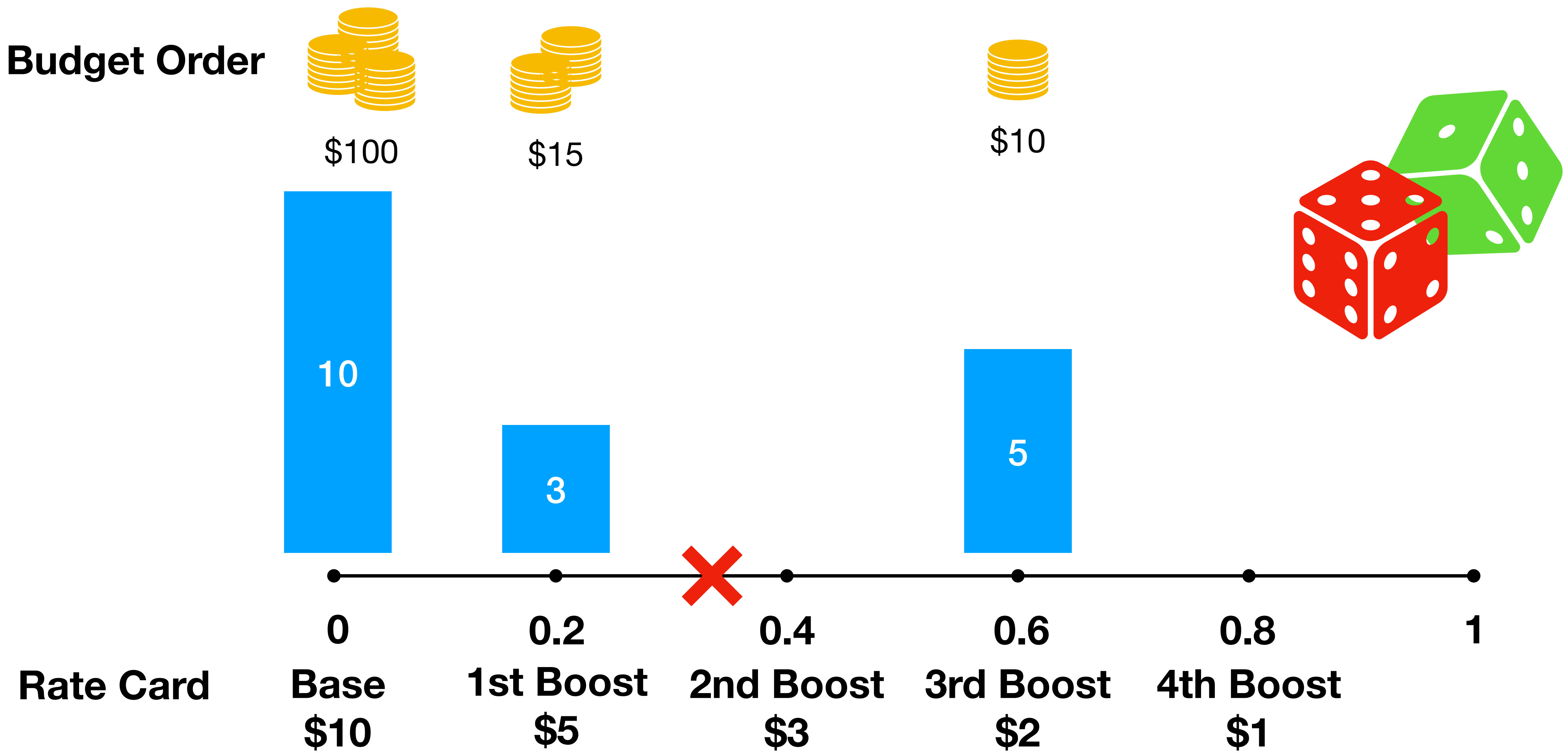
$100 $15 $10

	0	0.2	0.4	0.6	0.8	1
Rate Card	**Base** **$10**	**1st Boost** **$5**	**2nd Boost** **$3**	**3rd Boost** **$2**	**4th Boost** **$1**	

Ladder Pricing

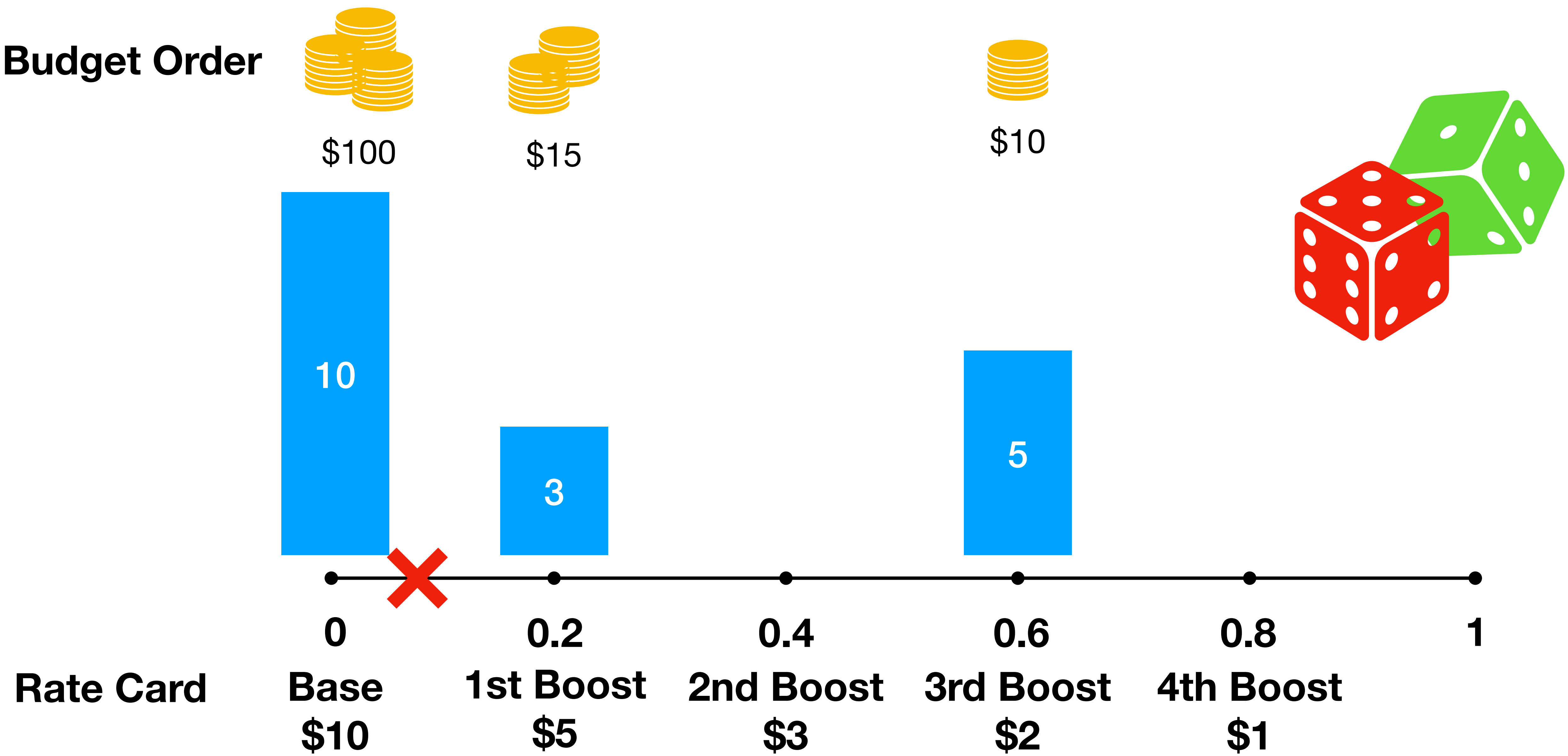


Ladder Pricing



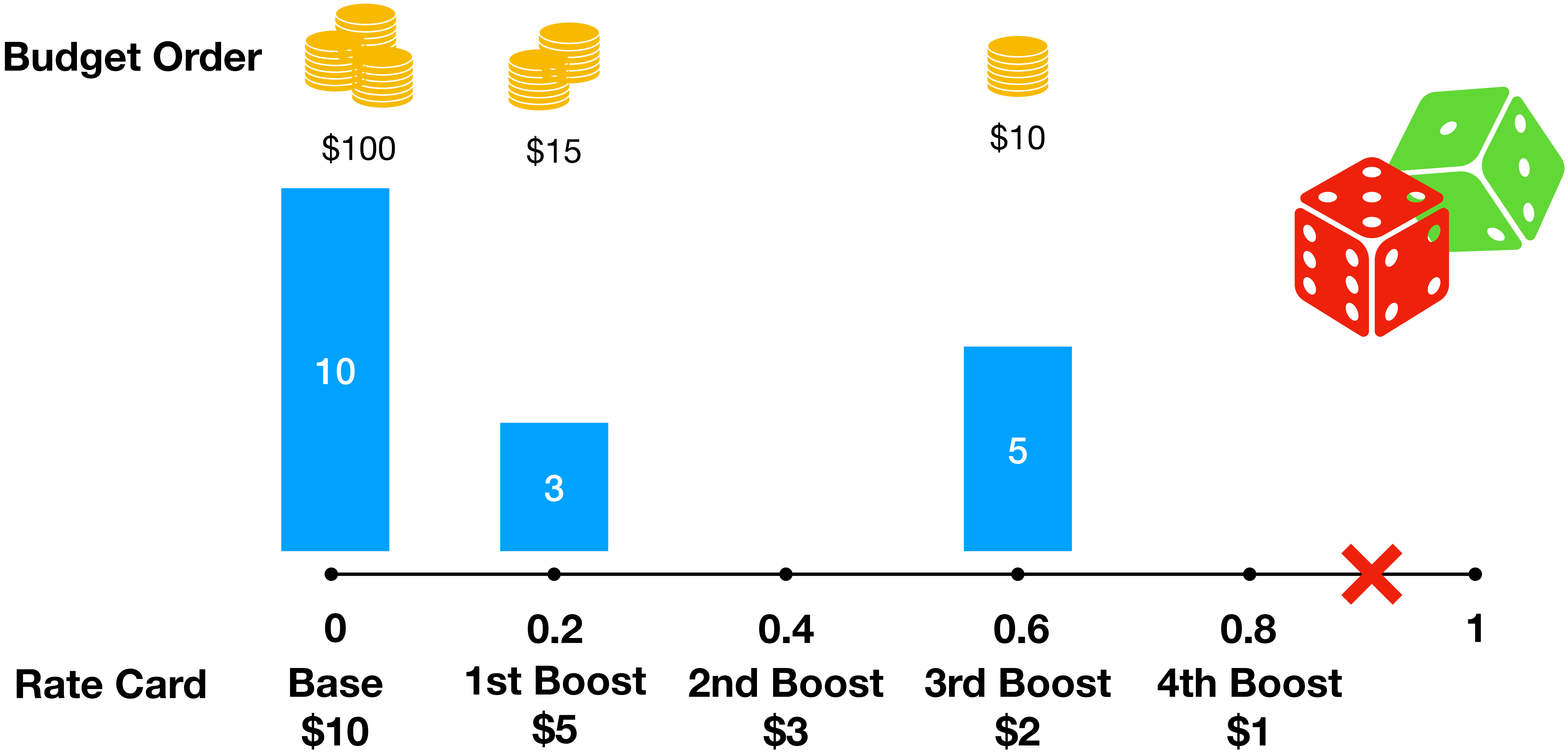
You get 13 units of bandwidth

Ladder Pricing



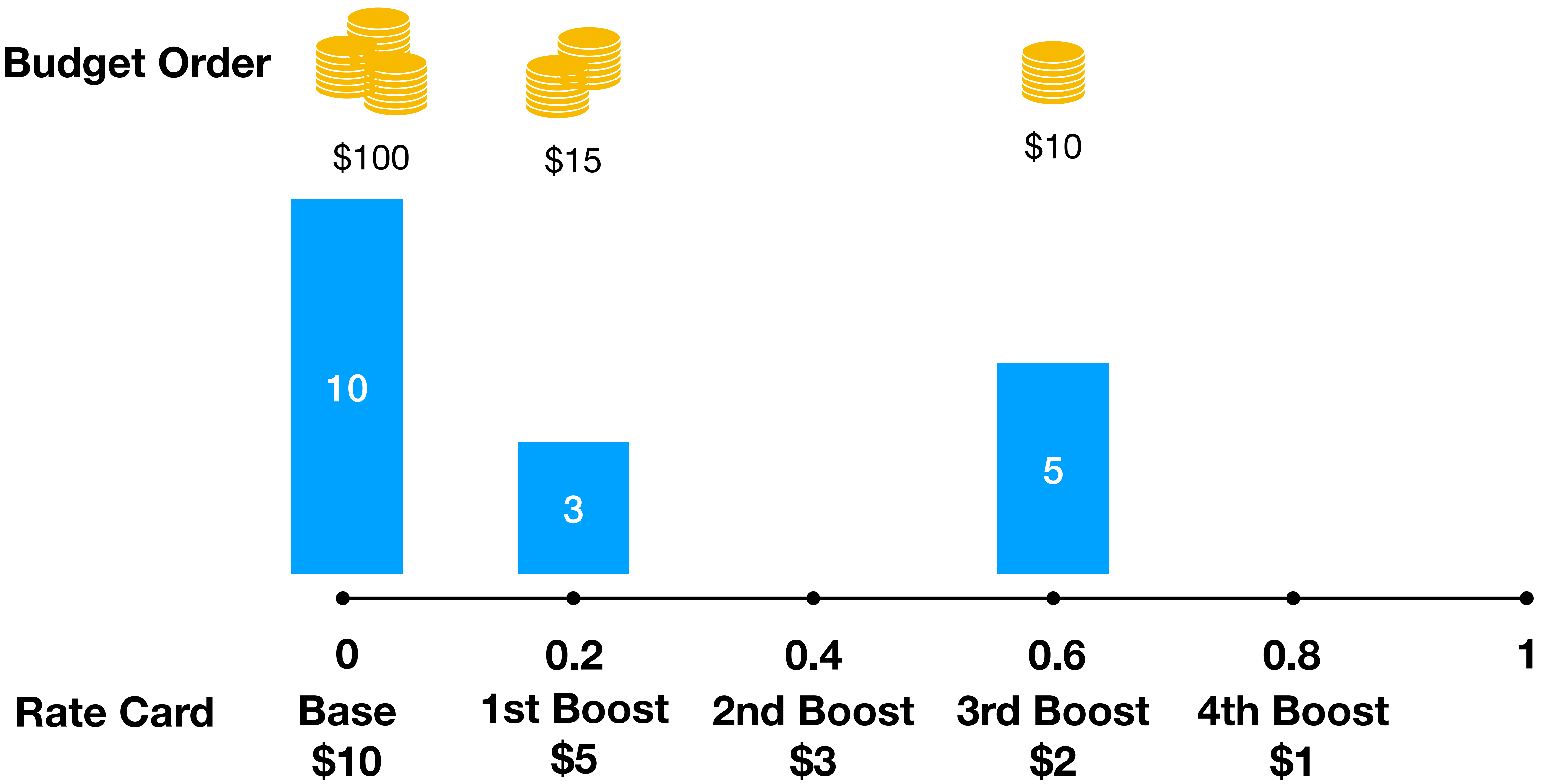
You get 10 units of bandwidth

Ladder Pricing



You get 18 units of bandwidth

Ladder Pricing



$L =$

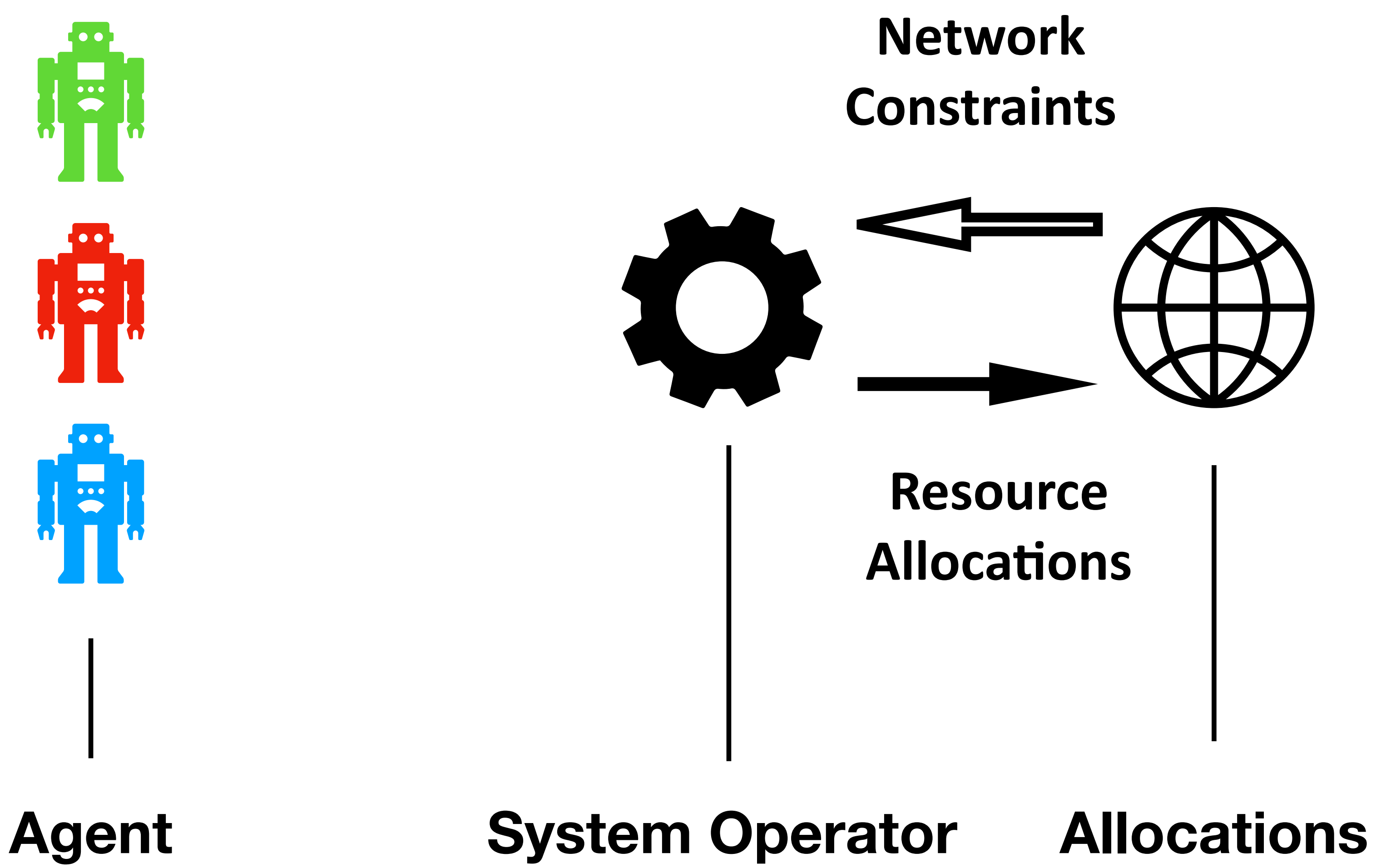
probability	40%	40%	20%
outcome	18 units	13 units	10 units

Agent Gain

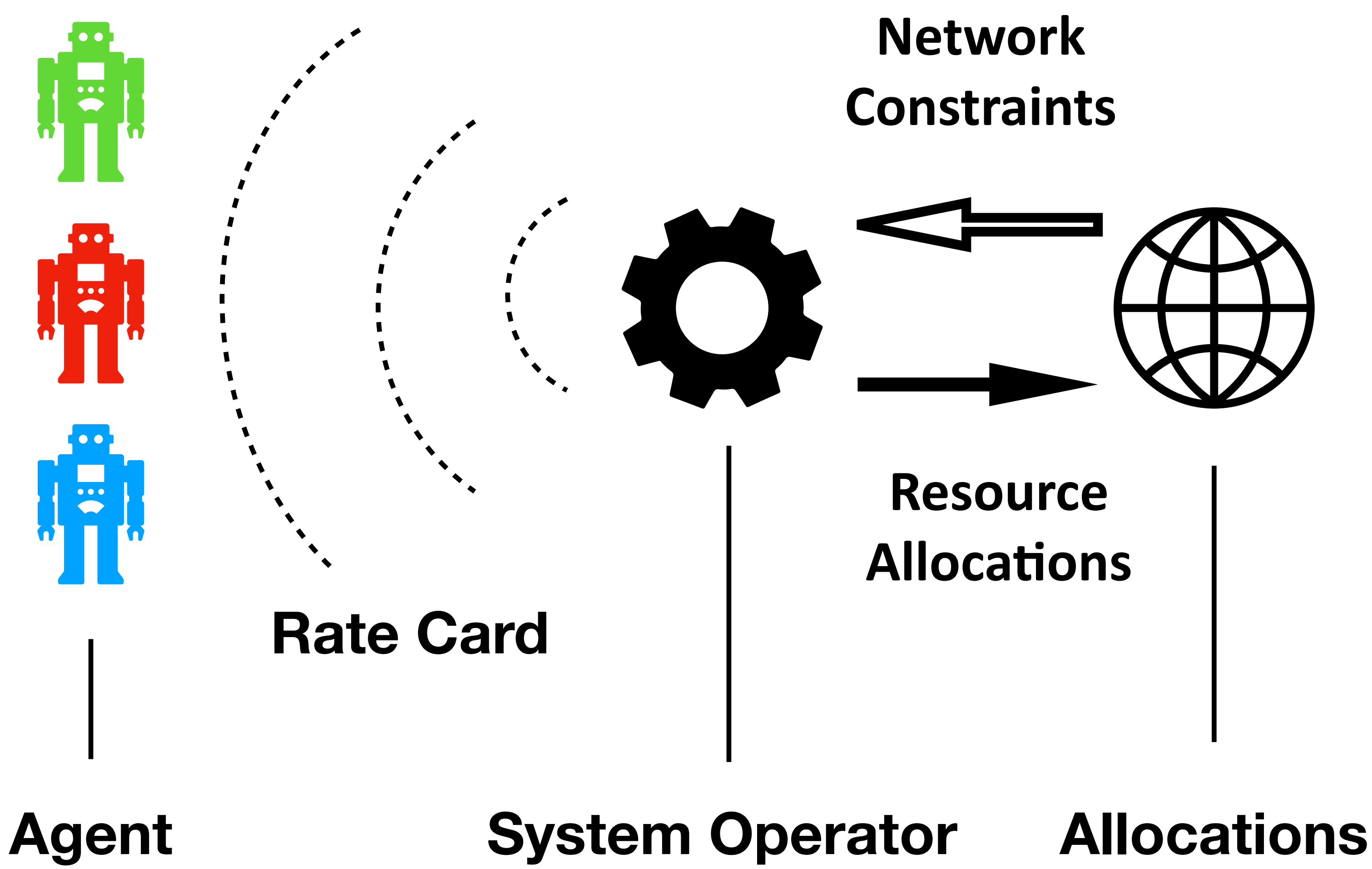
$V(L) - 125$

CPT value of lottery minus total budget

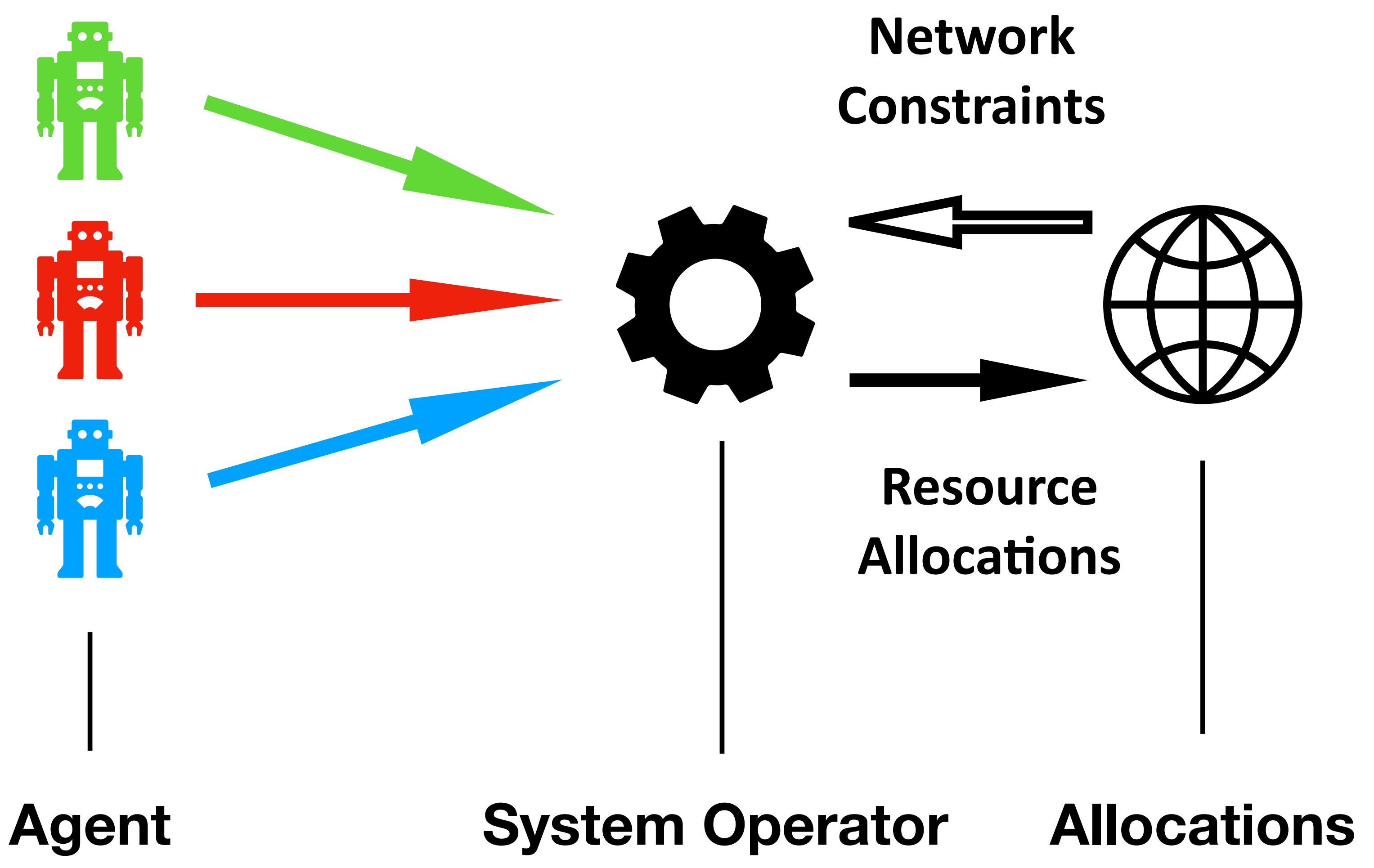
Agent-Operator Decomposition



Agent-Operator Decomposition

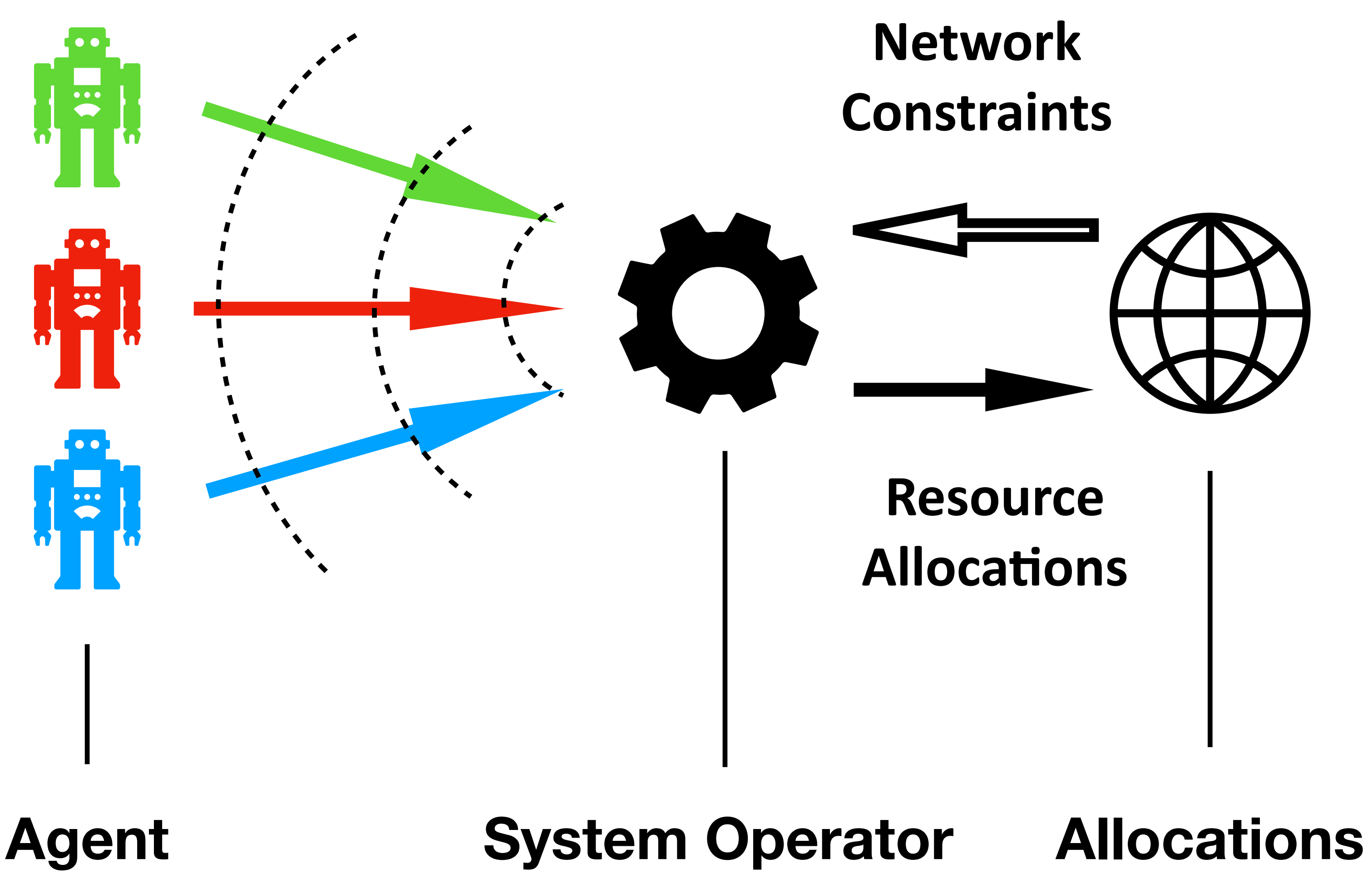


Agent-Operator Decomposition



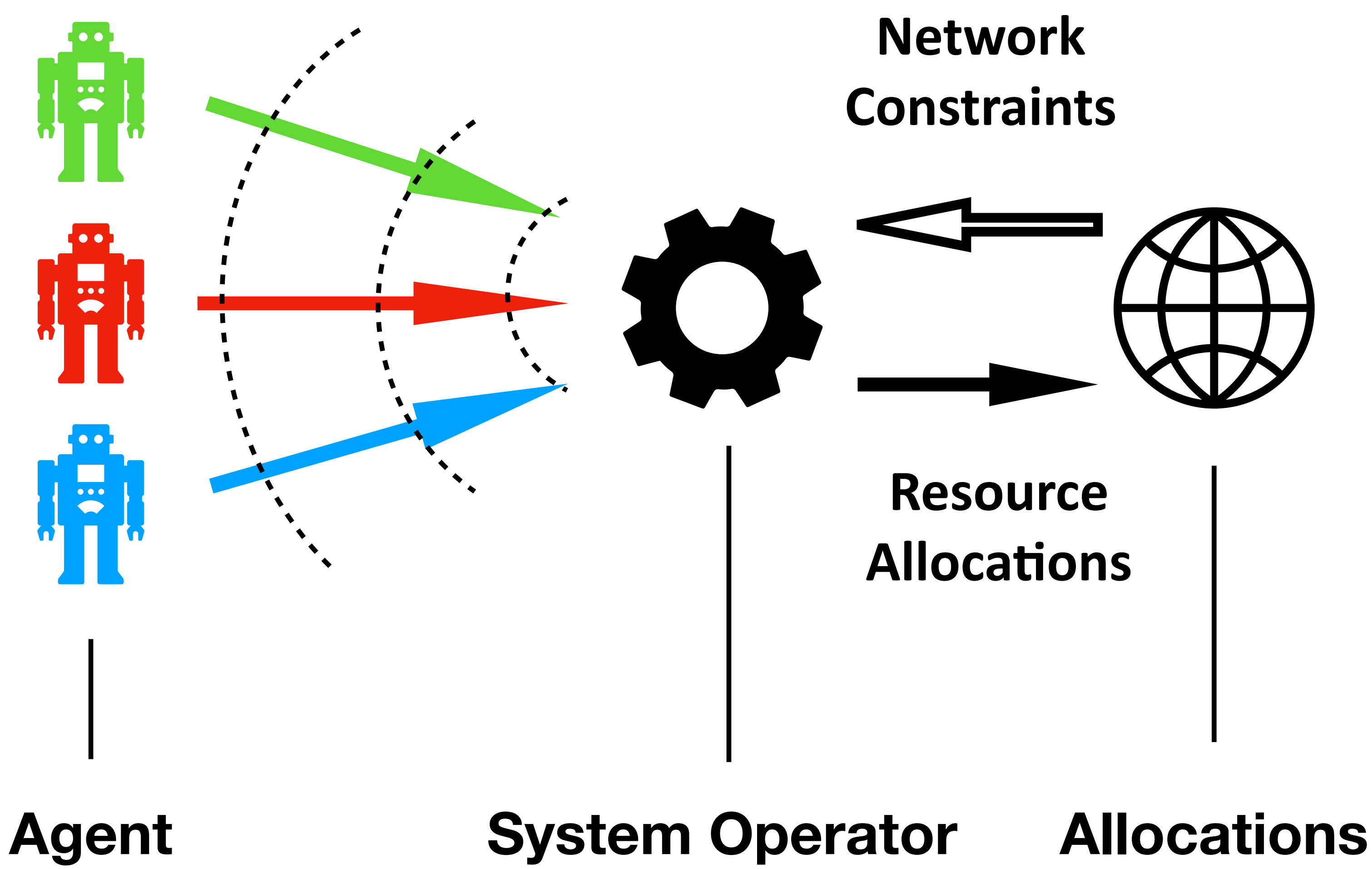
Each agent solves agent problem using preference information and current rate card and submits optimal budget orders.

Agent-Operator Decomposition



System Operator collects all budget orders, solves operator problem and broadcasts updated rate card

Agent-Operator Decomposition



Theorem: At equilibrium system problem with a fixed permutation is solved optimally.

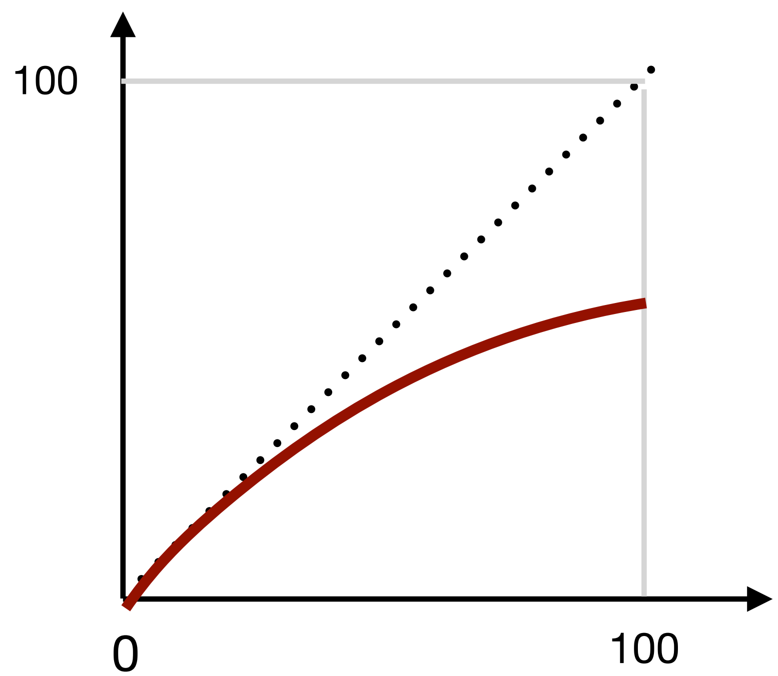
Using Lyapunov stability analysis you can show that the iterative process converges

[Phade, Anantharam. GameNets 2019.]

Example

100 agents, 1 link shared by all the agents
Link capacity = 100 Mbps

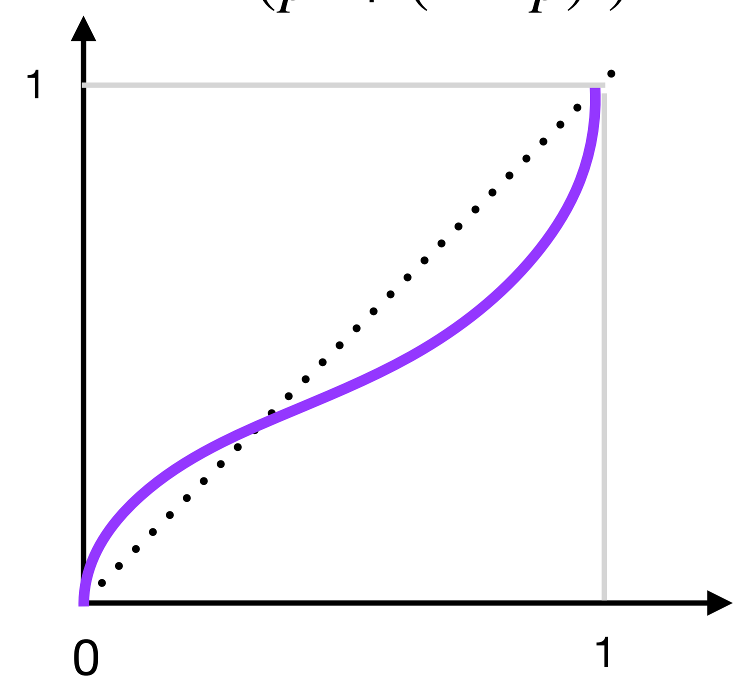
$$v(x) = \sqrt{x}$$



Value function

Deterministic Allocation
Social welfare = 100

$$w(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}, \gamma = 0.61$$



Probability weighting function

Lottery Allocation
Social Welfare 157.91

Winner gets 69.45 Mbps and the rest is equally divided amongst the remaining agents (0.31 Mbps)

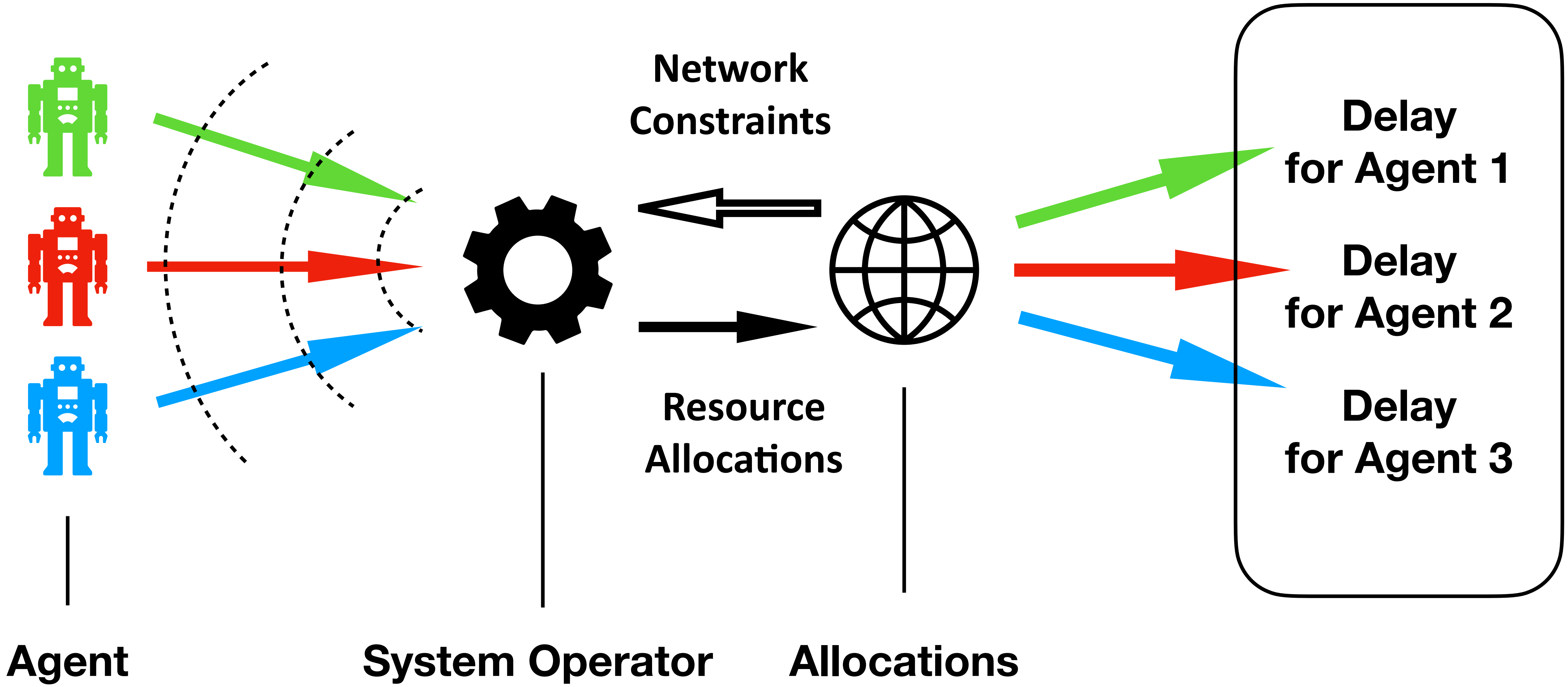
Relaxation

- Relax the permutation matrix to be doubly stochastic matrix
- Convex Opt. problem
- Equivalent to the problem where link constraints hold in expectation (Soft Constraints)
- Can be solved efficiently using market-based pricing

Application to Cloud Computing

- Today's pricing schemes are mostly one price for all job types
- Customers have varying preferences over their job delays
- Multi-tier service: Some jobs are served immediately while others are scheduled for later execution
- Scheduling problem as a resource allocation problem
- Treat tiers as links with capacity constraints
- This viewpoint is particularly well-suited for serverless computing

Agent-Operator Decomposition

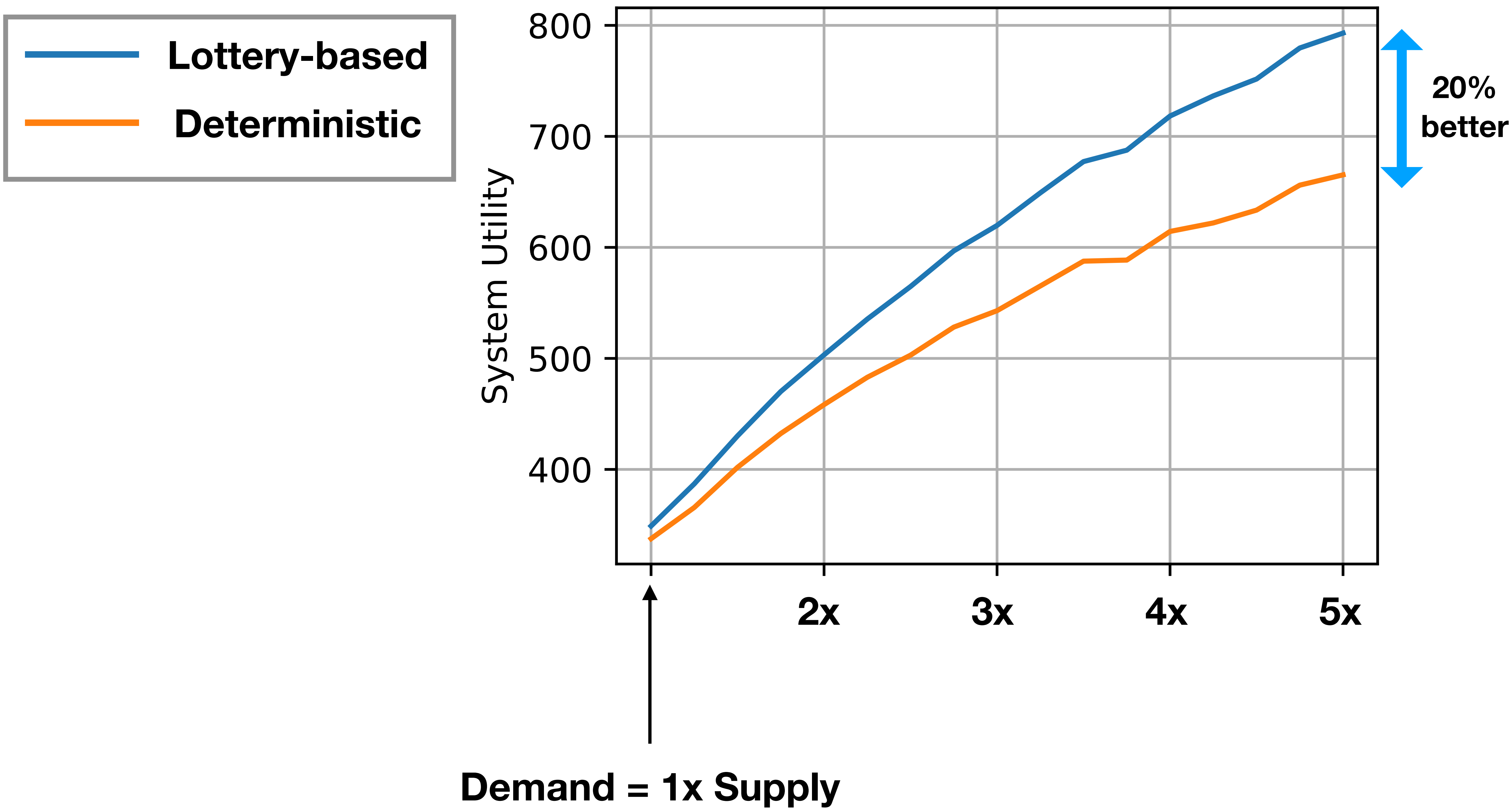


Agents have preferences over delays (or outcomes) and not on allocations

Under EUT it is okay to consider agents preferences on allocations directly. But not under CPT.

[Phade, Gupta, Courtade, Kannan. ArXiv 2020.]

Simulations



[Phade, Gupta, Courtade, Kannan. ArXiv 2020.]

Extending to other avenues

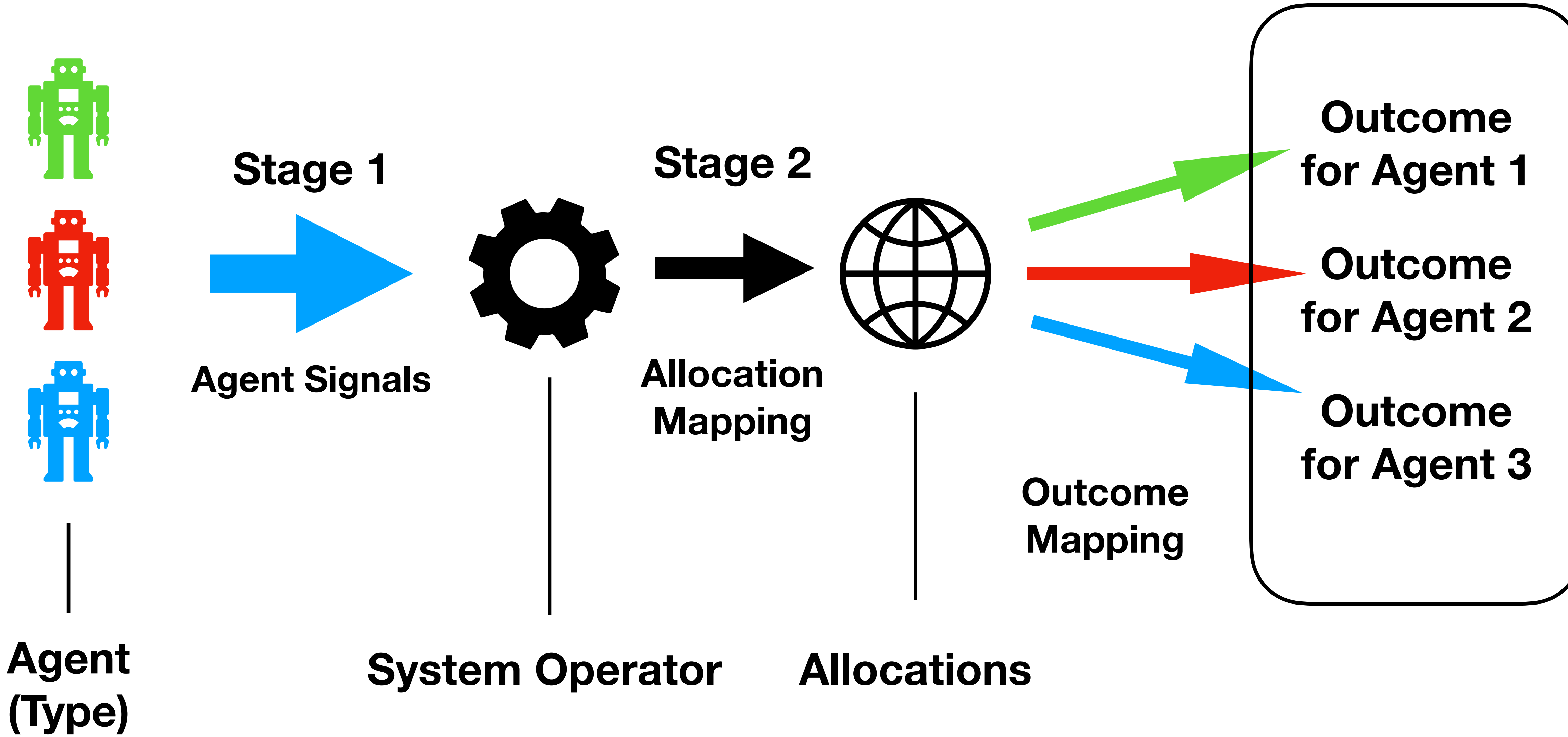
- Additional sources of uncertainties
  - lack of information about the outcomes
  - each agent has uncertainty and beliefs about other agents behavior

- Mechanism Design provides a general enough framework to study these settings

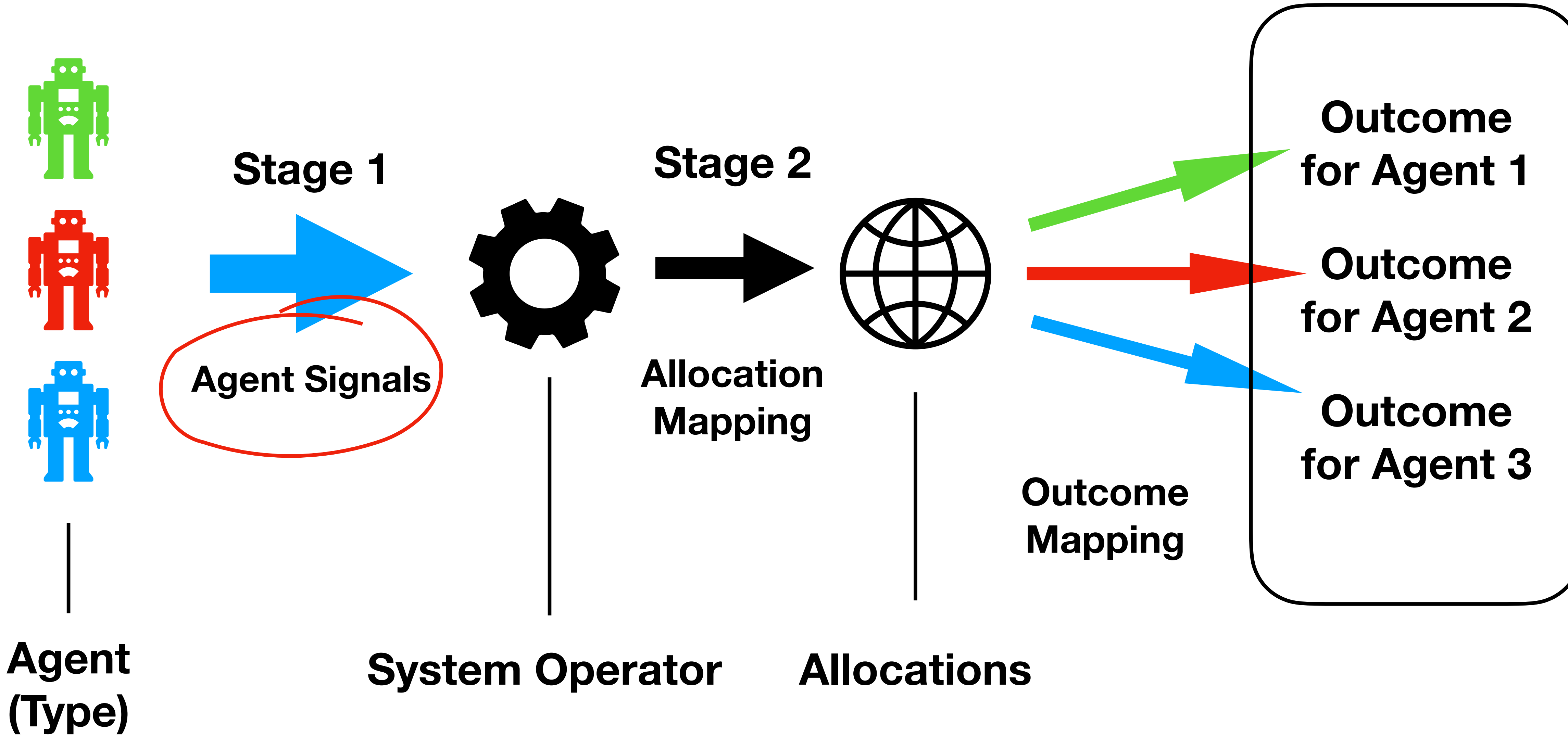
- Traditional applications: Auctions, Principle-Agent problems, Contract Theory, Housing and School allocation

- Modern day applications: Uber, Amazon, Google Ads

Mechanism Design



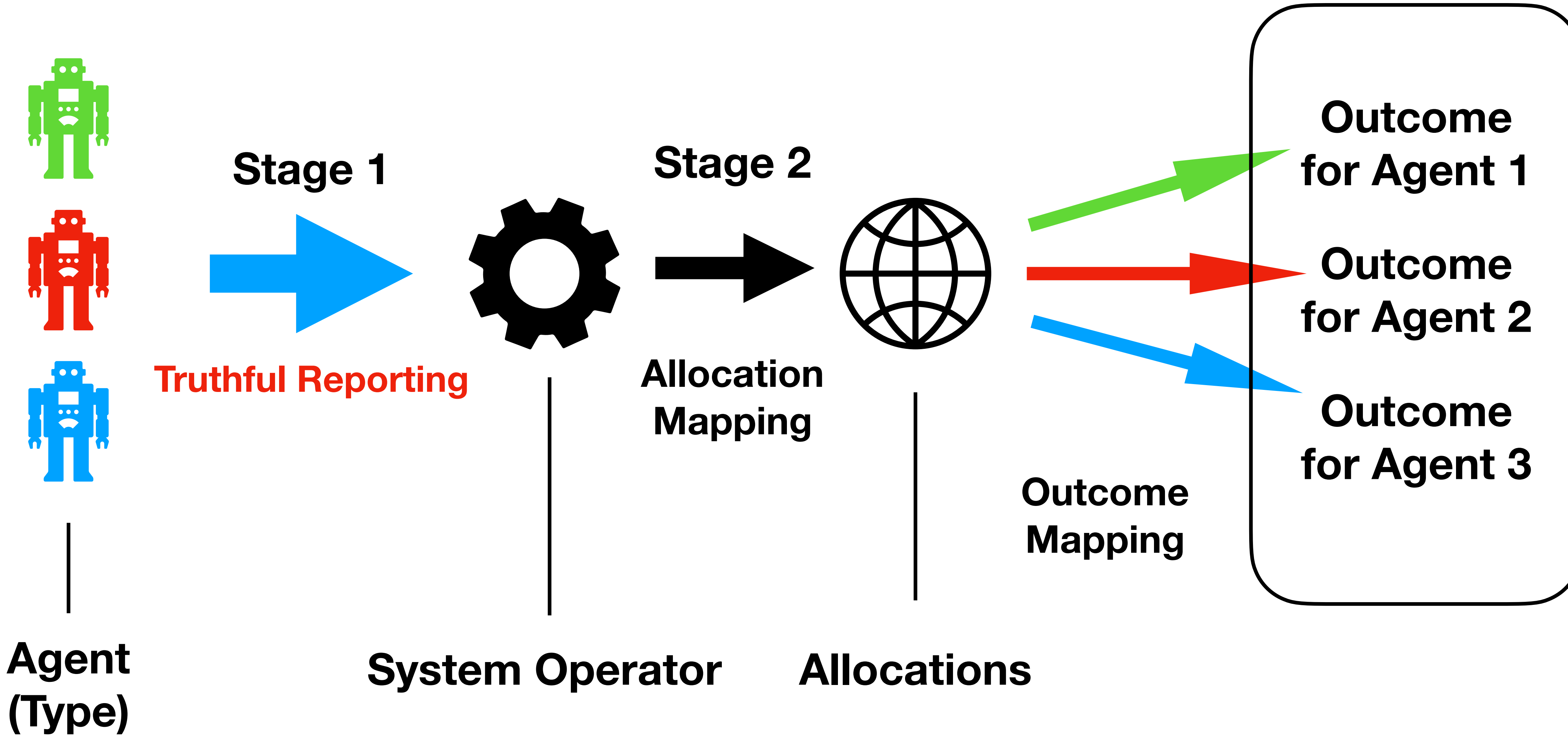
Mechanism Design



Social Choice Function

Agent Type Profile → Allocation

Mechanism Design

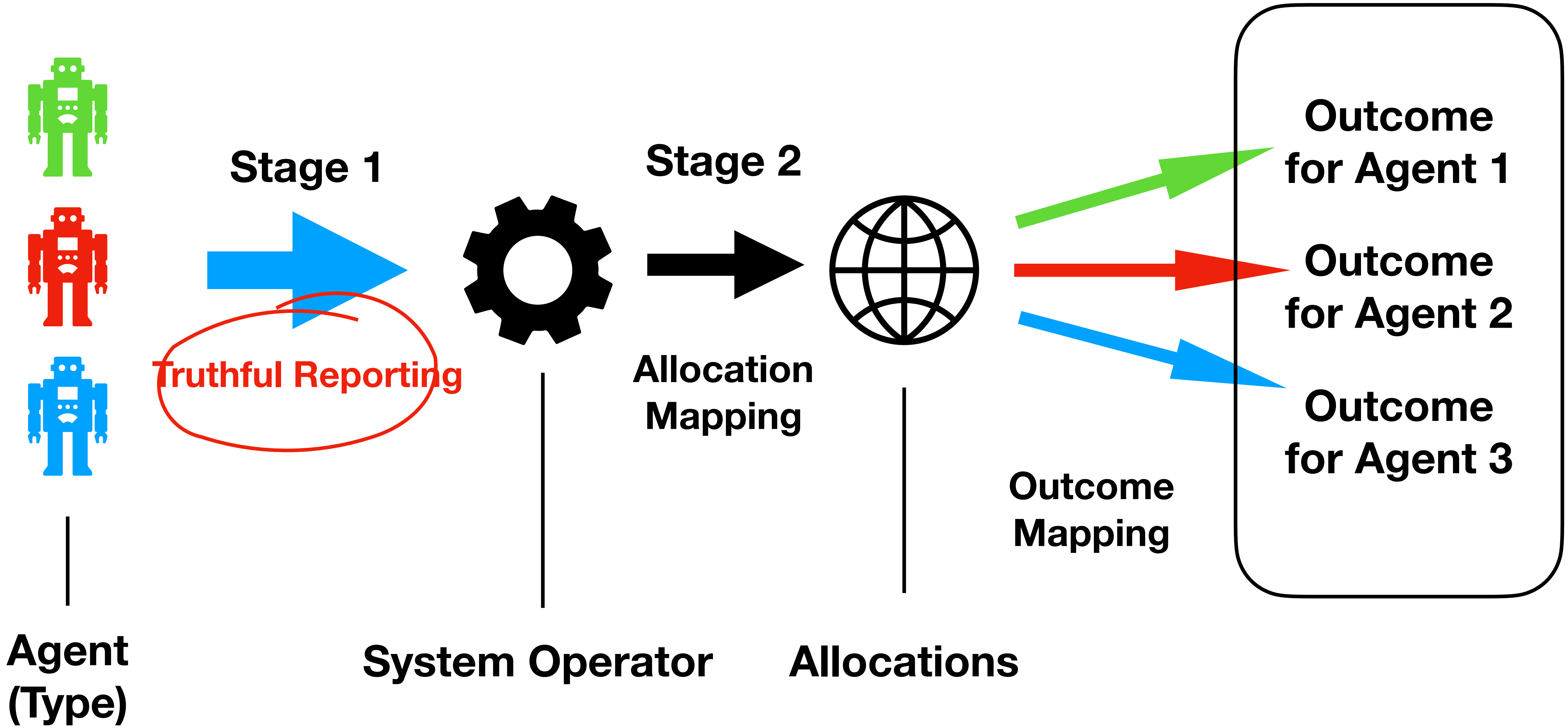


Under EUT

Revelation Principle (Myerson 1981)

- w.l.o.g. assume signal set = type set for each player
- restrict attention to direct truthful mechanisms

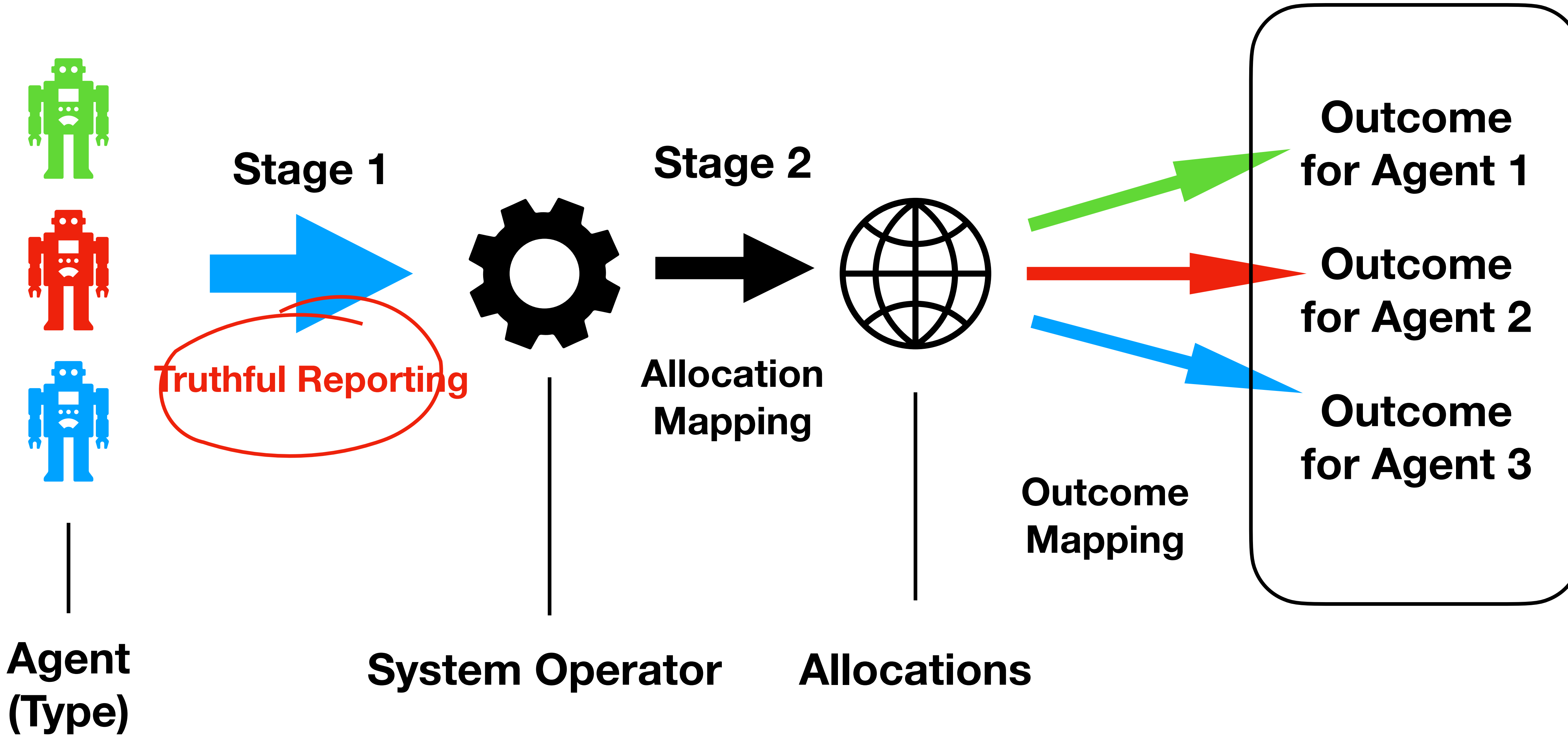
Mechanism Design



Importance of truthful strategies

- Limits on information availability
- Computational and cognitive limitations
- Users with different levels of access to information and computation.

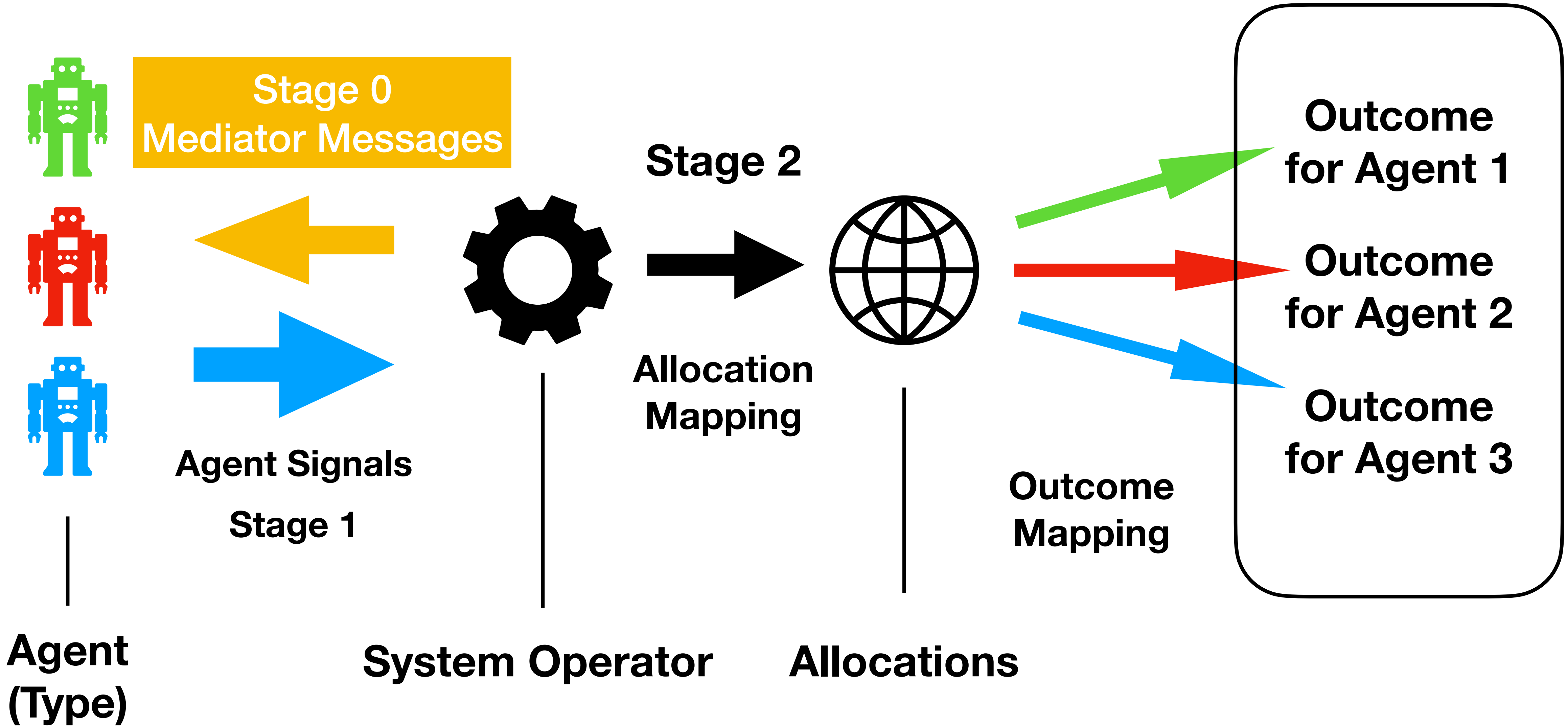
Mechanism Design



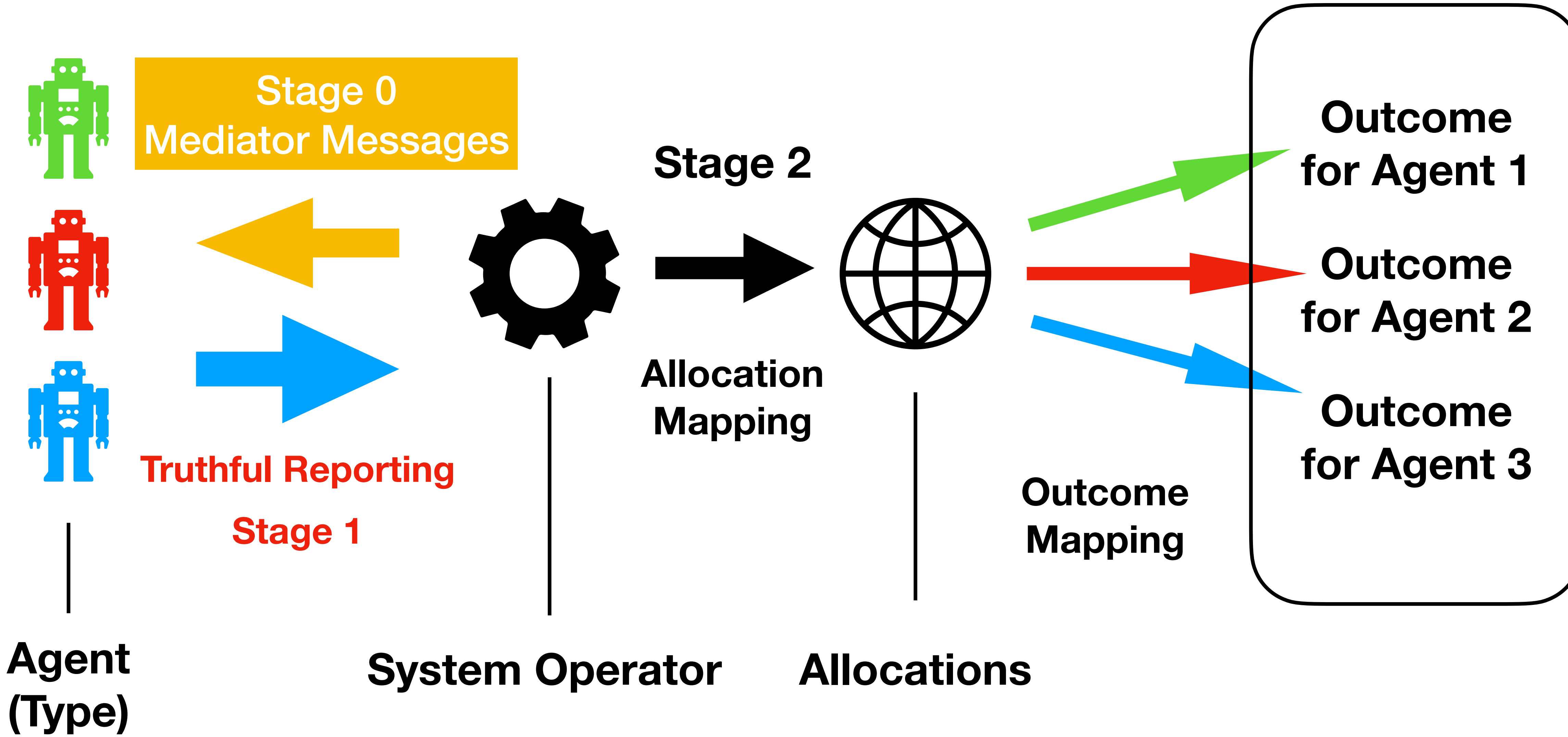
Revelation Principle does not hold under CPT

Previously observed to fail in second-price sealed-bid auctions (Karni and Safra 1989)

Mechanism Design



Mechanism Design



Revelation Principle (under CPT) recovered

- w.l.o.g. assume signal set = type set for each player
- restrict attention to direct truthful mediated mechanisms

[Phade, Anantharam. ArXiv 2021.]

Game Theory with CPT

Non-cooperative games with CPT players

- CPT Correlated Equilibrium
- CPT Nash Equilibrium

→ Blackbox Equilibrium

- Adopt definitions by Keskin 2017
- Establish geometric properties: CPT correlated set can be non-convex and disconnected
- Extend the result by Nau et al: Nash equilibria lie on the boundary of correlated equilibria

[Phade, Anantharam. Decision Analysis 2019.]

Blackbox Equilibrium

- Violation of Betweenness property
- Deliberate choice of randomized actions - Blackbox strategy
- New notions of CPT Nash equilibrium

[Phade, Anantharam. ArXiv 2020.]

Mediated games with CPT players

- Mediated CPT Bayes-Nash
- Mediated CPT Correlated Equilibrium

→ Revelation Principle

- Mediator sends private messages to each agent drawn from a joint distribution.
- Agents observe these signals before playing.
- The Bayes-Nash equilibrium of this game gives rise to an extension of correlated equilibrium notion.
- Appropriate notion for learning in repeated games

[Phade, Anantharam. DGAA 2021.]

Revelation Principle

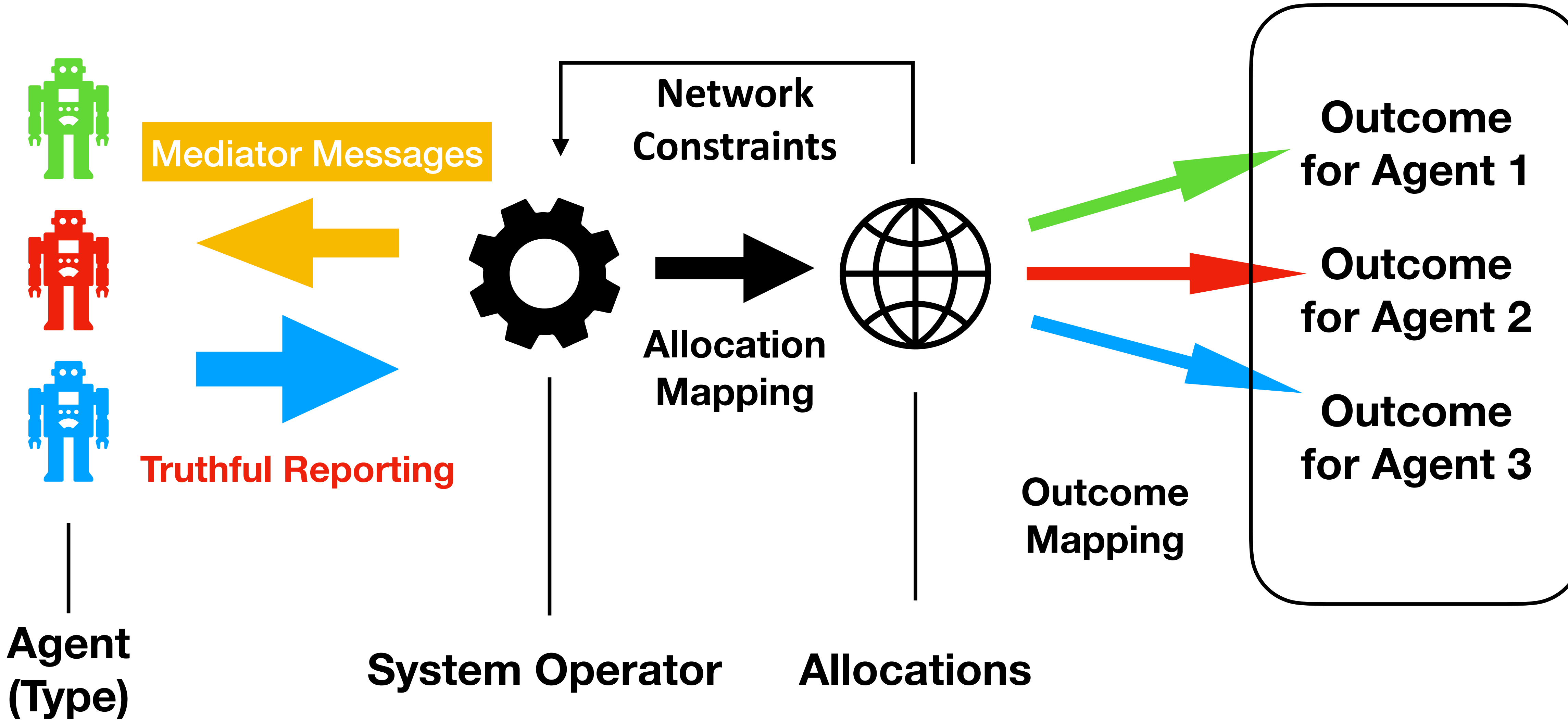
- Borrowing the idea of mediator to mechanism design we get this result.

[Phade, Anantharam. ArXiv 2021.]

"If we can agree that the economic problem of society is mainly one of rapid adaptation to changes in the particular circumstances of time and place, it would seem to follow that the ultimate decisions must be left to the people who are familiar with these circumstances, who know directly of the relevant changes and of the resources immediately available to meet them. We cannot expect that this problem will be solved by first communicating all this knowledge to a central board which, after integrating all knowledge, issues its orders. We must solve it by some form of decentralization. But this answers only part of our problem. We need decentralization because only thus can we ensure that the knowledge of the particular circumstances of time and place will be promptly used. But the 'man on the spot' cannot decide solely on the basis of his limited but intimate knowledge of the facts of his immediate surroundings. There still remains the problem of communicating to him such further information as he needs to fit his decisions into the whole pattern of changes of the larger economic system."

–Friedrich Hayek. "The use of Knowledge in Society." (1945)

Role of Communication, Data-Analytics, and AI in Network Economics



- Compare mediator messages with menus shown to customers: available options, corresponding prices, and associated uncertainties.
- Recommendations that are aware of resource constraints and market conditions
- Often solutions coming from theory involve complex signals.
- Repeated interactions leading to data-driven approaches. Using AI to learn preferences and assist decision making and signaling.

Fairness and Ethical Considerations

- Utilitarian approach, i.e. maximize social welfare. CPT allows us to capture perceived happiness.
- Reference point dependence allows us to capture social norms and expectations in groups.
- Lottery-based framework provides a chance for participants with limited budgets even under peak-price conditions.
- Do lotteries try to exploit the agents?

Takeaways

- CPT is a useful tool to capture human preferences in network settings
- Lotteries provide a rich environment and can be useful
  - A mystery solved: traditional theory explains the use of lotteries only when the goods to be allocated are indivisible. But lotteries are used much more generally.
- Agent-Operator decomposition is an important way to solve network economics problems.
- ML/AI can provide ways to make these frameworks practical and much more advanced

Thank you for your attention!

Thank you!

Advisor



Committee





Collaborators

Thank you!

BLISS Lab

Thank you!